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STRATEGIES TO DEVELOP EFFECTIVE PROBLEM SOLVING HABITS FOR ENGLISH LEARNERS IN A PROBLEM-BASED LEARNING CLASSROOM

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by

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CHAPTER ONE

The Challenge of Word Problems for English Learners

Overview of Chapter

There are few tasks that elicit anxiety in a math classroom more than word problems (VanSciver, 2009). Math teachers of English Learners (ELs) face the challenge of helping their students solve these anxiety-producing problems in a language that may be uncomfortable for them, and help them identify a valid solution method. This raises the question: *how do students with varying levels of English proficiency respond to identified teaching strategies noted in the research literature that support them with developing a "problem-model approach" to solving mathematics word problems?* (Hegarty et al., 1995, p. 18) The purpose of this study will be to test strategies from the research literature, specifically Read-and-Think (RAT) Math, to assist 7th graders with varying levels of English language proficiency (ELP) to interrogate mathematics word problems, and to observe how these 7th graders respond to said strategies.

Anxiety from solving word problems may be related to the multi-faceted nature of the activity described by several researchers (Hegarty et al., 1995; Hohn & Frey, 2002). For example, according to these authors, to effectively solve a word problem, one must be able to do the following:

- competently read each sentence, perceive the relationships between the variables being described,
- build some mathematical representation of the story or situation, devise a solution plan, execute that plan, and
- finally, interpret the solution in its original context, checking to ensure it makes sense.

With such a complex group of skills involved, it is little wonder that solving story problems is particularly challenging for ELs, particularly when these problems require culturally-specific background knowledge, refer to abstract concepts like interest, or include irrelevant information and/or language that does not clearly signal what operation to use (Kim et al., 2015).

Considering how difficult word problems can be for ELs, many educators, including Clement and Bernhard (2005), Dick, Foote, White, Trocki, Sztajn, Heck, and Herrema (2016), Heater, Howard, and Linz (2012), Hohn and Frey (2002), Griffin and Jitendra (2008), and Orosco (2014), have developed a wide variety of strategies to help them solve them. These are patterns of thinking that are explicitly taught, which students then apply to solving word problems. Some are published in books and educational journals, while others, such as those described later in this chapter, are spread teacher-to-teacher, either through conversations or non-academic online sources. Many of these strategies share the goal of helping students make sense of a math problem. Hegarty et al. (1995) refer to this process of making sense of a problem as having a “problem-model approach” (p. 18).

Other word problem strategies, although generally not those found in academic literature, teach students to look for "key words" as a shortcut (Clement & Bernhard, 2005). In contrast to the problem-model approach, Hegarty et al. (1995) refer to this process of using key words to translate written language into a mathematical expression as following a "direct translation approach" (p.18). In this researcher's experience, the direct translation approach often leads to students using invalid heuristics for selecting a strategy, even if those students are quite skilled with performing the calculation. For example, I have watched diligent, but procedurally-minded students find the word *each* in a problem, circle it, and immediately begin multiplying the numbers in the problem, even if *each* was signaling division, or had nothing at all to do with signaling what operation to use.

This chapter will explore the background of this conundrum for the researcher and its relevance. The next section will illustrate my history of teaching students with varying levels of ELP, how my experiences have shaped my thinking around how best to serve them, and how those experiences have developed my desire to research this question to benefit my students. Secondly, "Potential Importance of the Thesis Question" will justify why I believe this inquiry to be a worthwhile endeavor, and explain the potential benefit to fellow mathematics teachers of multilingual students. Finally, "Outline of the Rest of the Capstone" will break down the structure of the following chapters.

Importance of Capstone Inquiry to the Writer

When I first began teaching in 2014, I found myself working at a middle school where the vast majority of the students were the children of Somali refugees and, similarly, the vast majority were considered ELs. In order to help meet their needs, teachers were trained in the Sheltered Instruction Observation Protocol (SIOP) (Echevarria et al., 2000) and were expected to follow certain practices outlined by the model. The expectations of SIOP included having both content and academic language objectives for every lesson, building background knowledge for students before introducing the main concept of the lesson, and giving them the chance to practice reading, speaking, listening and writing in every lesson.

In my experience, however, the SIOP model was frequently shortchanged for the sake of maintaining classroom control and achieving short-term gains on standardized tests. Most SIOP trainings were punctuated with anecdotes of times teachers attempted to use one of the best practices from the training only to have to abandon it when manipulatives became projectiles, and partner discussions became off-topic shouting matches. A combination of "low" students (as many students who appeared to be slow to understand academic content were constantly and referred to by the staff at the school) and rebellious behaviors pressured my colleagues to resort to using traditional direct instruction, particularly in math classes. This method of teaching, which will be further examined in Chapter Two, involves the teacher explaining and modeling how to solve a problem, and then gradually releasing the students to practice the teacher's method (Hudson et al., 2006). Direct instruction is not necessarily incompatible with SIOP, but the way it was implemented at this school strongly emphasized the teacher's voice at the expense of student voice.

The excuse for this teacher-centered approach was that the students "couldn't handle" hands-on, inquiry-based activities, and that context would simply confuse them, rather than help them make sense of mathematical principles. I was told multiple times by administration and instructional coaches that my most of my students were too "low" to make sense of math and needed to be taught using the "I Do - We Do - You Do" methods pushed in Doug Lemov's (2015) book *Teach Like a Champion*, popular among administrators, especially in charter schools. It was at this school that I was introduced to the "CUBES" method of solving word problems.

"CUBES" is an acronym for "Circle the numbers, Underline the question, Box the key words, Evaluate and Solve", and we were told to teach it to the students. The phrase "box the key words" sent a very misleading message to the students, and most had thoroughly embraced the direct translation approach to solving word problems described above. Again and again, I saw students at this school box the word *and* and immediately start adding two numbers, or box the word *of* and immediately begin multiplying without any regard to the quantitative relationship in the story. For example, students had to solve the following word problem on the first test I wrote:

A team of 7 bank robbers steal $25,903.50. They decide to split it up evenly. How much money does each bank robber get?

I was astonished to see some students multiply the two numbers, even with the deliberate inclusion of the phrase "split it up evenly". I continued to teach the CUBES strategy for the sake of consistency, by the time I left the school in June of 2015, having taught there for one year, I was becoming apprehensive of the strategy's benefits for my EL students.

My second and third year of teaching took me to another SIOP school, this time one with a mostly Hmong population. However, unlike with the previous school, behavior was not generally a problem, and there was no such pressure to ignore the practices laid out by the SIOP model. Again, not wanting to impose a whole new strategy for solving word problems on the students, I talked to the fifth grade math teacher to find out what strategy she used, which she called “SKATE”. Like CUBES, I cannot find any reference to this strategy in academic literature, but seems to have been created and spread by teachers.

This time, the acronym stood for “Survey the problem, Keep the important information, Attempt to estimate, Take your time to solve, and Examine your answer.” This time, there was no reference to key words, so I hoped that my students would be able to adopt a problem-model approach to solving problems. After reviewing the strategy with my students, however, it became clear that they were well familiar with key words and used the same direct translation approach I had encountered before when solving word problems. It was unclear where they had picked up this habit, but some teacher, tutor or family member had conveyed it along the way.

Eager to get my students to pay attention to the relationships between the quantities in word problems instead of just the key words, I personally designed a series of lessons meant to force the issue. In these lessons the word problems had been scrupulously scrubbed of obvious key words, such as *less*, *increased* or *divided*. Removing these key words forced students to determine what the information in the problem *meant*, and how it related to the operation. I had noticed earlier that my students were quick to understand problems about increases or decreases, but struggled with part-part-whole relationships. An example of a part-part-whole relationship is having 73 fish in a pond (the whole), 39 of which are trout (one part) and 34 of which are koi (other part). These were the relationships we focused on first.

In particular, my students seemed to have trouble understanding that when they were told the whole and asked to find one of the parts, that *adding* the numbers they were given was an invalid strategy. When we explored multiplication and division relationships, students would write down the number of groups, the size of the groups and the total number of objects, observe which piece of information was unknown, and use that to decide whether it was a multiplication or a division problem. These lessons were met with mixed success; many students were able to complete the lessons with some difficulty, but struggled to recall or apply the concepts again later.

It quickly became apparent that many of the students were unaccustomed to making sense of the quantitative relationship in the problem, and lacked the number sense to recognize which operation was at play. For example, one problem I had students solve was:

Kou has a fish tank that holds 7½ gallons of water. He is using a jug that holds ⅝ gallons. How many jugs of water will it take Kou to fill up his fish tank?

To solve this problem, students had to recognize that 7½ gallons was the total amount of water, that ⅝ was the size of each smaller jug, and that they were trying to find the number of ⅝ gallon jugs it would take to equal 7½ gallons. Then they had to recognize that knowing these two numbers and searching for the number of jugs was a division situation. Without a clear keyword, my students were unable to identify the correct operation, and many simply stalled and waited for help. As helpful as I believed these lessons to be, it was also clear that spending two weeks on operational sense in sixth grade was no substitute for learning to make sense of the operations in elementary school, or having a curriculum that constantly spiraled it in.

This school sent me to two professional developments focused on math so I could harness new, researched-based ideas in my classroom. The first was a Guided Math workshop, which introduced me to creative new uses for manipulatives, such as base-10 blocks, fractional pattern blocks, small group instruction methods, and, at long last, confirmation that teaching students to seek out key words was not merely an unhelpful strategy, but a counter-productive one for children's thinking (Boonen, de Koning, Jolles, & Van der Schoot, M., 2016; Clement & Bernhard, 2005; Dick, et al., 2016; Griffin & Jitendra, 2008; Van de Walle, 2014; Van der Schoot, Bakker Arkema, Horsley, & van Lieshout, 2009; Verschaffel, De Corte, & Pauwels, 1992). The second workshop they sent me to was focused around Jo Boaler's (2016) freshly published book *Mathematical Mindsets*.

While at the training, I asked some veteran math coaches what their preferred word strategies were for ELs, and they told me about the Read-And-Think (RAT) method, in which students read a problem one sentence at a time, make sense of each sentence before moving on to the next, and try to infer what the question is going to be before they read it. I eagerly began pioneering RAT with my math intervention students in small groups, and was encouraged by the success it seemed to have, especially when combined with encouraging the student to represent the situation with a drawing as he or she read the problem. I had one student in particular who struggled with reading and was prone to making random guesses in math class. This student suddenly became deeply engaged with trying to predict what question I would ask him and coming up with a clear diagram that helped him select a reasonable strategy for solving the problem.

Unfortunately, this school underwent a period of severe turnover and internal conflict. During my second year, the authorizer sent a letter of intent to close the school and 75% of the staff left within the span of a year, and I was no exception. I then spent the 2017-2018 academic year at an ill-fated Project Based Learning school that did not survive its pilot year. Finally, in 2018, I found myself at another school primarily serving students from the Hmong diaspora. This school, while not a SIOP school, still serves a similar population with many ELs. In addition, this school had just adopted the *College Preparatory Mathematics* (CPM) (CPM Educational Program, 2013) curriculum, which focuses on building students' number sense and conceptual understanding through collaborative, problem-based learning and spiraling content. At last I had found a school that prioritized conceptual understanding over procedural learning for its ELs. While the change has been welcome, it has, of course, presented a significant challenge in that my students are constantly interacting with word problems without necessarily having the skills to break down the text independently.

With these experiences behind me and the present challenge before me and my colleagues, the question of how best to teach ELs to analyze word problems has never been more pertinent. In the remainder of this thesis, I will attempt to contribute to our understanding of the effectiveness of different word problem strategies. As long as an untold multitude of teachers are relying on key word strategies, the question of how to replace these strategies with a more meaningful, relevant one will remain urgent.

Potential Importance of the Thesis Question

The question of how best to serve ELs in mathematics remains an open one. As I will explore in Chapter Two, there is a divide between those who advocate teaching students through traditional direct instruction and those who advocate more collaborative inquiry-based instruction. In 2008, the National Mathematics Advisory Panel declared that any claims that one approach should be thoroughly favored over the other are not supported by the evidence. That said, what is certainly clear in the literature is that the key word method is unhelpful in many situations, and may even be damaging to students' development as mathematicians. There is still much work to do, however, around identifying and testing approaches to helping ELs solve word problems *without* resorting to this method.

For my own practice, my math teaching colleagues and I are finding ourselves in unfamiliar territory. We are implementing a curriculum that was designed, seemingly, under the assumption that most of the students in the room would be able to read the questions and generally understand enough to start trying *something*. In reality, we are seeing students who can clearly decode (meaning sound out the words on the page) but struggle to make meaning out of the relationships. Many lack the skills to *interrogate* the text, that is, read the text with an analytic lens in order to obtain the desired information.

Much of our previous experience as educators has prepared us to clearly explain mathematics to students, but we are constantly discussing the struggle of getting our students to read a problem and then start attempting to solve it without constant "hand-holding". Students are meant to discover new mathematical concepts through solving these problems (CPM Educational Program, 2013); if they cannot get started solving the problems, how are they supposed to effectively learn the concepts? Having a greater understanding of what word problem strategies seem to be effective for ELs will enable me and my colleagues to grow in our ability to teach in this kind of environment.

Teaching ELs to interrogate a text to identify the quantitative relationships in it has never been more important to us as educators, now that they are expected to do this on a daily basis. We need a clearer idea of what practices are effective with ELs, and other students whose ELP hinders solving word problems. This knowledge will inform our teaching across the department, and could influence the practices of other teachers with similar student populations, as well as teachers using similar curricula to CPM.

With regards to the body of research, this study will seek to contribute more data to the general pool of knowledge about teaching ELs to solve word problems. My research will also contribute to our understanding of how ELs function in a classroom that uses Problem-Based Instruction (PBL) - that is, instruction in which students learn math concepts through struggling with problems (Jarvis, 2016). This type of instruction is further explained in Chapter Two. This study will also hopefully illuminate whether students shift to using a problem-model approach to solving problems over the course of the study: an area of research that needs further exploration, especially for ELs.

Outline of the Rest of the Capstone

Chapter Two will provide a review of the literature related to this study. It will first justify a key assumption of this study: PBL activities that encourage students to develop number sense are valuable and worth supporting. As such, the eventual data will be evaluated in terms of evidence of conceptual understanding over students getting a correct answer. The review of literature will then lay out much of the current understanding of the effectiveness of various word problem strategies and methods for teaching ELs mathematics. Chapter Three will present the methodology of this study, and Chapter Four will present and analyze the data gathered. Finally, Chapter Five will lay out my conclusions and the implications and limitations of the data.

CHAPTER TWO

Review of the Literature

Overview of Chapter

The challenge of teaching math effectively to English Language Learners (ELs) is a prominent and relevant one in today's education system. Of particular interest and difficulty is teaching ELs to surmount the complex task of word problem solving. One potential way to achieve this is through strategy instruction, particularly strategies that emphasize making sense of a problem, which is referred to as a "problem-model approach". This stands in contrast to a "direct translation approach", in which students attempt to use the *words* (instead of the relationships) in the problem to write an expression or an equation and solve it (Hegarty, Mayer, & Monk, 1995).

The purpose of this study will be to identify different teaching strategies that support 7th graders with varying levels of English language proficiency to interrogate mathematics word problems, and to observe how these 7th graders respond to said strategies. It will explore the question: *how do students with varying levels of English proficiency respond to identified teaching strategies noted in the research literature that support them with developing a problem-model approach to solving mathematics word problems?*

This chapter will explore four main topics relevant to this inquiry. The first section will examine the challenges that face ELs in mathematics, as well as techniques that various researchers and educators recommend to ameliorating these difficulties. This section will elaborate the conditions faced by mathematics teachers of ELs, which are the motivation for conducting this study. The second section will detail the educational philosophy and practices of reform mathematics teaching, as well as their potential benefits to ELs. The purpose of this section is to provide context for the study, as well as justification for pursuing word problem solving as a topic of inquiry. The third section will provide the theoretical foundations of the problem-model approach and its well-established superiority over both the direct translation approach and the related key word method. Lastly, the fourth section will lay out several prominent word problem strategies found in academic literature and discuss common themes among them.

Challenges Facing English Language Learners in Mathematics

Since the early 1980s, the number of English Language Learners (ELs) in K-12 American classrooms has dramatically increased; as of 2009, they made up 21% of all school-aged children (Kim et al., 2015). In that same time period, according to Wiest (2008), mathematics teaching has also become more literacy-based as educators increasingly emphasize classroom discussions and multiple representations of mathematics, including diagrams, manipulatives and verbal explanations (Siebert & Draper, 2008). To add to the increasing challenges mathematics teachers already face, Hoffert (2009) points out that ELs still must pass the same standardized tests as native English speakers.

This inevitably raises the question of how well these ELs fare in mathematics classrooms. The data is not encouraging. According to Kim et al. (2015), ELs overwhelmingly do not show grade-level mastery on standardized tests and struggle to progress at the same rate as their peers. Kim et al. (2015) go on to discuss the particular struggles of ELs of Southeast Asian origin, which are often ignored by researchers and teachers due to Asians' status as the "model minority", despite these students following the same trends as other ELs and dropping out of high school at alarming rates. Many of these students are put in mainstream classes, despite their English ability not being high enough to effectively read the texts they are given (Brown, 2007).

There are many potential barriers to ELs learning in today's mathematics classrooms. For example, Hoffert (2009) explains that many ELs are also refugees who may have experienced trauma before leaving their country of origin, referred to as Students with Limited or Interrupted Formal Education (SLIFE). Many also face cultural difficulties, finding themselves being asked for the first time to disagree or argue with classmates in a collaborative learning group (MacDonald et al., 2014). However, as one might expect, the most widely discussed barrier facing ELs is their difficulty in acquiring academic English.

It is important to recognize there is a difference between academic and conversational English. Many students are fluent in conversational English, but are still classified as ELs. According to Brown (2007), conversational English is the language used in informal situations, and ELs typically acquire it within two to three years. This is often referred to as basic interpersonal communicative skills, or BICS (Cummins, 2008). Academic English, on the other hand, is the language of academic texts, which students typically learn at school instead of home (MacDonald et al., 2014), or, as Cummins (2008) refers to it, cognitive academic language proficiency (CALP). ELs typically take five to seven years to acquire this version of English (Brown, 2007), but according to Collier and Thomas (1997), this can take as long as seven to ten years for students who did not have formal schooling before coming to the United States (as cited in Hoffert, 2009). Teachers of ELs must be aware of the difference between these two types of language proficiency, as well as the difference in time they take to master.

All subjects have their own version of academic English (MacDonald et al., 2014), and they have some common themes. In general, as Brown (2007) explains, academic English is more formal and expository in style, and uses much more complex sentence structures, linking together multiple ideas with different clauses within the same sentence. While these texts are difficult enough for native English speakers to learn to read, they are exceptionally challenging for ELs to master (Brown, 2007). Mathematics has its own style of academic English with its own features that are challenging for ELs, which the next section will explore.

Mathematics Register. The term “mathematics register” was coined by Michael Halliday by 1978, which he defined as "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings” (as cited in Schleppegrell, 2007). Ming (2012) characterizes this language with its use of symbols (such as “5”, “+” and “=”), very precise meanings for words (which might have other meanings outside of mathematics), and a very concise writing style. Chan (2015) notes that this register of language is difficult for even English-proficient bilingual students, and thus it is incorrect to assume that these students will automatically acquire or understand it without help. Part of the difficulty for some ELs, arises from not having language to express a mathematical idea in their native language, and therefore not having a common underlying proficiency in the concept being expressed to draw on in either language when learning the English word (Cummins, 2005). On the other hand, MacDonald et al. (2014) cautions against teaching this language for its own sake, asserting that children learn the patterns of mathematical language as the need arises as they use mathematics. However difficult it may be to learn the mathematics register, understanding it remains essential.

Schleppegrell (2007) explains that conversational English knowledge is insufficient to convey mathematical meaning, and using it to describe mathematical ideas can result in inaccurate statements. Hence, mathematics has developed various ways to describe information, including mathematical symbols, technical language, and visual displays, such as diagrams and graphs. Schleppegrell goes on to show that mathematical language is not arbitrarily complex, but essential to making and communicating these complex meanings.

The first key feature of mathematical language noted above is the unique use of vocabulary. Mathematical terms have very precise definitions (MacDonald et al., 2014), such as defining a rectangle as a parallelogram (a four-sided figure with two sets of opposite parallel sides) with four right angles, as opposed to simply a shape with four sides, as children often learn first. Some of these terms are technical, used almost exclusively in mathematics (e.g. equilateral, rhombus, sum, or isometric), which Chan (2015) explains makes them often difficult for ELs to understand due to a lack of previous exposure.

However, possibly even more challenging are words that have one meaning in everyday language but another, more precise meaning in math (Schleppegrell, 2007). For example, an EL might know the word “rate” in the context of giving a movie a good or bad rating, but be unfamiliar with the mathematical meaning. These are called “polysemous words”, which ELs find confusing when assigning meaning (Mitsugi, 2017), and which comprise a high percentage of the English lexicon. However, as Mitsugi also notes, these words typically have a “core meaning”, which unites the various definitions, and finds that teaching ELs the core meaning of a polysemous word can clarify the other, more derived meanings.

The mathematics register also presents challenges on the levels of clauses, sentences and paragraphs, as ideas are developed over the course of several clauses or sentences (Chan, 2015). As noted before, one trait of mathematical language on this level is how it compacts several ideas together into short, but very complex, sentences. Ideas that would be expressed in several sentences might be presented as a single sentence in mathematics (MacDonald et al., 2014). For example, consider the following sentence:

Find the area of a rectangle with a base of 6 cm, a perimeter of thrice that length, and a diagonal of 5.2 cm.

This sentence conveys an objective (*find the area*), the classification of a shape (*rectangle*), the length of the rectangle’s base, and the relationship between the base and the perimeter (*thrice*). Schleppegrell (2007) argues that students need help from teachers to unpack this dense information until they acquire the skill themselves. Furthermore, this sentence follows another characteristic of mathematical language noted by

Schleppegrell, that is, leaving important information implicit. In this case, the phrase "*a perimeter of thrice that length"* could be used to find the height of the rectangle, which is necessary for solving the problem, but never stated explicitly. One last characteristic of word problems in particular that is challenging for ELs is the inclusion of irrelevant information: the length of the diagonal (Kim et al., 2015).

In addition to constructing sentences and clauses very densely, even common words take on new subtleties in mathematics. Schleppegrell (2007) also details the difference between using the word "to be" for attribution and identification. Attribution refers to a hierarchical classification: for example, saying that "a square is a rectangle" means that a square is a specific kind of rectangle. It would be incorrect to reverse "square" and "rectangle" in this sentence. Identification, on the other hand, refers to equating two ideas: for example, saying that "the range of a data set is the difference between the greatest and smallest values" is simply defining "range". It would be perfectly acceptable to reverse "the range of a data set" and "the difference between the greatest and smallest values". Understanding when "to be" is being used for an attributive process vs an identifying process, and therefore whether or not the order of the sentence can be reversed, can be confusing.

Another way that some of the most common words in English are used differently in mathematics is how conjunctions take on more precise meanings (Schleppegrell, 2007). Conditional phrases using "if" and "when", as well as phrases using "and" or "or" take on new, logical meanings. For example, ELs may struggle with understanding that finding the probability of rolling a 2 *or* a 4 on a 6-sided die means *combining* the probabilities with addition.

One final characteristic of the mathematics register that Schleppegrell (2007) details is referring to processes as if they were objects. For example, addition, subtraction, multiplication, division, and exponents are all processes. Mathematics problems often refer to them with nouns like sum, difference, product, quotient, and square (as in "*two integers have a product of 12 and a sum of 7*"). This is not necessarily intuitive for students, let alone ELs. ELs reading phrases like this may struggle to connect this wording to the familiar actions of adding and multiplying numbers.

With so many potentially confusing features of mathematical language, the challenge of helping ELs succeed in mathematics may seem overwhelming. That said, there has been plentiful research into what classroom practices benefit ELs described in the next section.

Effective Mathematics Instruction for ELs. Teachers can have a profound impact on ELs and their ability to acquire mathematical language and concepts. According to Chan (2015), when teachers explicitly draw students' attention to the ways a word problem presents information and mediates the process of unpacking it, ELs can start to develop these skills for themselves. Similarly, Schleppegrell (2007) notes how teachers can clarify the precise usage of mathematical language for their ELs by rephrasing the students' explanations using correct language: a technique called "recasting". Beyond teacher mediation, structuring classes to give ELs maximum opportunities to communicate about mathematics promotes the development of mathematical language and ideas.

According to Hoffert (2009), it is essential for ELs to speak, read, and write about mathematics every single day. Allowing ELs to collaborate and communicate with their peers gives them more opportunities to use mathematical language and develop both mathematics and language skills (Wiest, 2008). This collaborative work can be particularly helpful when ELs are paired with both a native English speaker and a fellow EL within a group, so that they hear correct language usage being modeled in English (Wiest, 2008; Hoffert, 2009). Being able to discuss the content in their home language also helps them develop their common underlying proficiency in the concepts, scaffolding their learning the English terms (Cummins, 2005). In particular, having ELs solve challenging problems in a collaborative setting allows them to learn mathematics more deeply (Wiest, 2008) and may lead to higher performance than they would achieve in a traditional class (Garrison et al., 2007).

Another way to engage ELs in this type of mathematical conversation is the think-pair-share strategy, as pointed out by Ming (2012), in which students think of an answer to a question, discuss their ideas with a partner, and then share with the whole group. Schleppegrell (2007) cautions, however, that in these settings, students may still need teacher mediation to express their ideas in language that is mathematically correct. For example, a student may explain in a think-pair-share that they know they can find the area of a shape by multiplying its base and height because it is a *square*, when they really mean that it is a *rectangle.*

How a teacher frames the lesson can be helpful as well. ELs in particular benefit when time is taken to build their background knowledge and connect it to the activity (Echevarria et al., 2000; Wiest, 2008). It is also important to give directions using clear annunciation, a comprehensible speed, simple sentences, gestures, and vocabulary the students already understand (Echevarria et al., 2000; Hoffert, 2009). Hoffert (2009) also stresses the benefits of making sure every lesson involves some cumulative review, so that ELs can continue to master and recall skills and concepts they have already learned.

Lastly, many authors and researchers write of the benefits of providing ELs with visual aids and manipulatives. Pictures, graphs, and diagrams help ELs understand the context of a problem (Wiest, 2008), and they give much of the information to an EL without as much burden being placed on understanding the text (Chan, 2015), and also scaffolds students' ability to visualize the problem and, thus, think abstractly. Being able to visualize a math problem allows students to make their problem solving process more concrete and easier to connect to their lived experiences (Ming, 2012). Similarly, having students take notes in a graphic organizer may help them recall steps to procedures, possibly due to their visual and organized nature (Hoffert, 2009).

With all this in mind, it is actually possible that the mathematics classrooms of today, with their increased focus on communication, might be beneficial for ELs, despite the challenges they pose. The next section will explore the potential benefits of reform mathematics instruction for ELs.

Traditional vs Reform Curricula: Implication for English Language Learners

In 1989, the National Council of Teachers of Mathematics (NCTM) published its *Curriculum and Evaluation Standards for School Mathematics*. In the following decades, according to Sayeski and Paulsen (2010), a significant divide between two predominant math teaching philosophies has persisted among researchers, teachers, administrators and politicians alike. According to these authors, much of the focus has been put on different curricula embodying the two philosophies, traditional curricula and reform math.

Traditional curricula emphasize practicing computation skills, memorizing math facts (Hanson-Thomas, 2009), implementing standard algorithms, and teaching a single topic to mastery before moving on to a new one (Sayeski & Paulsen, 2010). Hudson et al. (2006) refer to the philosophy behind traditional curricula as either "explicit teaching" or "direct instruction", which they use interchangeably within the same work.

Hudson et al. describe explicit lessons as "teacher-delivered and structured," and generally progress through the following stages. In phase one, the teacher reviews previous material to activate the students' prior learning, to help them connect their prior knowledge to the new material. In phase two, the teacher demonstrates how to solve a problem, providing metacognitive "think-alouds" and questioning students to maintain engagement and to assess understanding. Lemov (2015) famously nicknames this phase as the "I do" phase of a lesson, which is often how educators refer to this phase colloquially. The teacher then proceeds to guided practice, in which the teacher guides the students through solving similar problems themselves, gradually releasing the students to more and more independence. Lemov similarly nicknames this as the "we do" phase. Finally, students are released to independent practice, in which the students practice solving the problems fully independently, or the "you do" phase.

On the other side of the divide is "reform" math, the approach generally championed by the NCTM (Hudson et al., 2006). According to Hanson-Thomas (2009), these curricula tend to favor student interaction and problem solving. They emphasize conceptual understanding over procedures, and view learning as socially-constructed, rather than transmitted by a teacher (Hudson et al., 2006). Sayeski and Paulsen (2010) describe how reform curricula often employ a wider variety of algorithms and a spiraled structure, in which one topic is not fully mastered before moving on, but is drawn on repeatedly thereafter to reach mastery over time. An example of a multiplication algorithm used in reform curricula because of its emphasis on conceptual understanding is the "generic rectangle" (also known as the "area model"), demonstrated below on the left, while the "standard algorithm" is demonstrated on the right (Figure 1).

25(47) demonstrated in two contrasting multiplication

Generic Rectangle

	40	7
20	20(40) = 800	20(7) = 140
5	5(40) = 200	5(7) = 35

```
   800
   200
   140
+   35
  1175
```

Standard Multiplication

```
   47
x  25
  235
  940
 1175
```

Figure 1. Two contrasting multiplication algorithms. Created by the author, 2018.

The instructional approach that underlies most reform curricula is called "Problem-Based Learning" (PBL), also referred to as "Problem-Based Instruction" (Gasser, 2010; Inglis & Miller, 2011), which the remainder of this section will discuss in further detail. Jarvis (2016) characterizes PBL as an approach to a lesson in which students learn mathematical concepts by cooperatively struggling through problems in groups.

Just as explicit instruction has a generally accepted progression for a lesson, PBL lessons generally follow three stages (Jarvis, 2016). First, the teacher presents and possibly demonstrates the activity students will be completing. Secondly, the students perform the activity, which is usually centered around discovering or exploring a mathematical concept. For example, in the *College Preparatory Mathematics (CPM)* curriculum (CPM Educational Program, 2013), students "discover" that 7 - (-2) can be rewritten as 7 + 2 by solving several subtraction problems using manipulatives, then looking for patterns among the equations. Finally, the teacher leads a debriefing session, in which the concept is formalized and any misconceptions that may have arisen during the activity are addressed.

For example, in a debriefing session around the integer subtraction lesson referenced above, students would share their ideas about how to efficiently subtract integers. Some examples might include "You can change subtraction into addition", or "Subtracting a negative can turn into a positive". It is the teacher's role to acknowledge the correctness of the students' ideas (both of the above statements are trying to express a valid observation), recast their statements to clarify ambiguous wording and misconceptions ("subtracting a negative *has the same effect as adding* a positive"), and ask probing questions to highlight cases that students might have overlooked ("Could we also rewrite subtracting a *positive* integer using addition?) to help students flesh out their understanding.

It should be noted that research comparing the two approaches to math teaching is mixed. Notably, in 2008, the National Mathematics Advisory Council found that both teacher-centered approaches (i.e. traditional curricula, explicit instruction) and student-centered approaches (i.e. reform math, PBL) are both beneficial in some situations and reject the idea that one approach should be used to the exclusion of the other. For example, explicit instruction has found a great deal of support in special education research, while reform practices, like collaborative learning groups, are shown to have positive impacts, particularly in the elementary grades.

In particular, some researchers (Hudson et al., 2006; Sayeski & Paulsen, 2010) point to the benefits of explicit instruction for students with mathematics learning disabilities. The aforementioned researchers accept that reform math is a dominant paradigm in mainstream math education, and explore ways of integrating explicit instruction into reform classrooms to benefit these special education students.

One blended approach suggested by Hudson et al. (2006) are structuring reform lessons in terms of beginning with concrete activities involving manipulatives, then moving to representing the idea with pictures, and then finally moving on to abstract representations (i.e. numbers, expressions and equations). This is often referred to as the Concrete-Representational-Abstract (CRA) approach. In this approach, the lesson might still follow a reform structure, but there would be opportunities for the teacher to explicitly guide students through the progression of representations.

Another blended approach suggested by Sayseki and Hudson (2010) involves a number of ways to integrate the two approaches. First, a teacher can provide some direct instruction to clustered groups of students with learning disabilities. Secondly, the teacher can explicitly teach effective strategies for solving the problems in a PBL lesson. Finally, the teacher can work direct instruction into the debriefing part of the lesson, often creating anchor charts to codify the knowledge gained in one lesson so it can be referred to in later lessons.

While the debate between these two philosophies is inconclusive, there is significant research detailing the benefits seen in classrooms that embrace PBL. The next section will examine these benefits, particularly focusing on its benefits to ELs.

The Benefits and Challenges of Problem-Based Learning. Several researchers, particularly those in other countries that make wider use of PBL, have observed significant benefits to the practice. In short, advocates of PBL claim and have shown that students participating in PBL become better at problem solving and understand mathematical concepts better (Gasser, 2011), significantly improve in mathematical discourse (Inglis & Miller, 2011) and learn to take responsibility for their own learning (Jarvis, 2016).

One reason PBL is worthy of attention is because of the success it has seen internationally, as revealed in test data. As Jarvis (2016) explains, PBL was first developed as a concept by Dr. Howard Barrows, a medical professor at McMaster University, who was influenced by the ideas of John Dewey in the 1970s. Despite PBL being developed in North America, Asian countries like Japan notably make a wider use of it. Gasser (2010) points out that these countries also tend to outperform the United States in the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA). Another country that has made use of PBL is Canada, where a 2011 study by Inglis and Miller saw standardized test scores for students in Ontario significantly rise after implementing a PBL program. As many countries that use PBL seem to see positive results, it is important to examine the ways in which PBL may have contributed to them.

Gasser (2011) points to the deeper level of thought required in a PBL classroom as the source of most of the benefits; processes like applying a strategy to an unfamiliar problem or making an argument why one solution is correct over another are generally more mentally rigorous than following procedural steps. Generally, according to Gasser, traditional classroom teaching in the United States is quite procedural and therefore asks less of students than countries making wider use of PBL; often American teachers demonstrate how to solve a problem without letting the students try to solve it on their own. This potentially deprives students of opportunities to develop conceptual understanding and therefore transferability of these concepts to new situations.

Furthermore, Gasser goes on to assert that teachers dominating the discussion in a classroom may harm students' chances to learn or compare problem solving strategies with each other. In a PBL classroom, two teams might attempt to solve the same problem two different ways: for example, a diagram vs a numerical expression. The teacher might direct these teams to present their methods to each other, allowing both teams to verbalize their own thinking, while also seeing another way to represent the same mathematics.

Another benefit of a PBL approach is the promotion of a growth mindset and reduction in math anxiety. Jo Boaler (2016), a collaborator of Carol Dweck's, characterizes a growth mindset as the belief that hard work increases one's intelligence. Boaler also indicates that this belief leads students to embrace mistakes as part of the learning process, persevere when confronted with difficulty, and grow dramatically more in mathematics achievement than those who believe intelligence is fixed and innate. Inglis and Miller (2011) found that after participating in a PBL program, their Canadian students became more willing to persevere in the problem solving process, rather than give up or resort to a direct translation approach, and adopted a more productive attitude towards math, gaining confidence and producing much more thorough work. Gasser (2011) points to practices in Taiwan and China in which students share their thinking with the class (a vital fixture of PBL), students understand that mistakes are part of the learning process, and that they will get the right answer in the future. Gasser speculates that the competitive culture of the United States might contribute to American students' reluctance to accept mistakes as helpful to their learning.

Finally, and most importantly to this study, PBL practices result in dramatically increased student discourse about math; that is, students in PBL classrooms spend a significantly greater time talking about mathematics. In another Canadian study, Jarvis (2016) found that students using reform curricula dramatically increased the amount of communicating mathematics to one another (which was accompanied by a boost in engagement), and, consequently, increased their retention of the math they learned. Jarvis explains that in a PBL classroom, the teacher acts as a coach to the students, and less of a presenter, therefore making space for students to talk with each other, rather than primarily listening to the teacher. Inglis and Miller (2011) similarly saw students improve their ability to use math vocabulary and explain their problem-solving process. They saw a particular rise in their ability to communicate about math through speaking, while recording their ideas in writing remained more of a struggle.

As PBL's benefits arise from getting students to productively struggle, it is hardly surprising that implementing it can come with its own struggles, especially early on. Jarvis (2016) noted that teachers transitioning to teaching with PBL tend to go through a "messy" phase in which they must struggle with several barriers: teachers' lack of deep content knowledge, pressure to teach all the standards, and a perception of inefficiency around PBL.

The first common barrier teachers face is their own lack of deep mathematical knowledge: a barrier which Celedón-Pattichis (2010) notes is especially notable for ESL teachers teaching math. To remedy this, Celedón-Pattichis points to quality and sustained professional development as an essential measure to ensure students, particularly ELs, are getting an equitable math education when starting to implement a reform curriculum.

Another barrier both Jarvis (2016) and Celedón-Pattichis (2011) noticed was that teachers feel pressure to get through all of the standards they are required to teach in time for standardized tests. Sometimes these teachers will even temporarily abandon their PBL-based curriculum to prepare for these tests. Finally, Jarvis (2016) noted that teachers often feel that PBL activities are less efficient than a traditional lesson (even though these same teachers find them more meaningful), and they are prone to worrying about classroom management concerns as students may begin misbehaving during the longer stretches of group work that are often present in a PBL lesson.

However, once this "messy" phase is over, the benefits of meaning-making are ready to harvest (Jarvis, 2016). This process of discourse and making meaning is exactly what makes PBL beneficial to ELs.

PBL and ELs: Increasing Student Verbalization. There is a limited amount of research available about how ELs perform in a PBL setting; in total, I found less than 10 studies addressing this. However, most of the research and theory appears to support that ELs benefit from productive struggle (the experience of attempting different approaches to solving a problem without being shown how first), which is a vital part of a PBL classroom, and the increased level of discourse in a PBL classroom (Hansen-Thomas, 2009; Lynch et al., 2018; MacDonald, 2017).

As mentioned before, one of the most important ideas in PBL is that of productive struggle. Lynch et al. (2018) list evidence that students who struggle productively in math are engaged in the process of making meaning of mathematics, and outperform students who learn surface level procedures. MacDonald (2017) argues that the ability to struggle productively, solve problems in a team setting, and think critically are not just essential in a math classroom either, but in the 21st century workplace as well. ELs are not exempt from this; in fact, they particularly need deep learning experiences, such as those provided by a PBL lesson (MacDonald et al., 2014).

Lynch et al. (2018) lament that, unfortunately, many teachers attempting to increase access for ELs do so by decreasing the rigor of the lesson. While it may be helpful to provide accommodations for an EL to help them make sense of a word problem (such as paraphrasing a problem or providing a picture), it is unhelpful to do so by decreasing the richness of the activity. This deprives ELs of an essential learning opportunity, because when ELs are engaged in the process of making meaning of mathematics, they simultaneously strengthen their ability to make sense of the English language (MacDonald et al., 2014), which is worthwhile in and of itself.

Possibly the most vital benefit of all for ELs engaged in a PBL classroom is the way in which they use the language of mathematics. In a traditional, explicit, teacher-centered classroom, as explained by MacDonald (2017), teachers will commonly interact with students using the IRE model: the teacher asks a question (teacher *Inquires*), to which the student *Responds,* which is followed by the teacher *Evaluating* the student's response. This often limits the students' level of verbal engagement to one-or-two word responses, hardly demonstrating a deep level of understanding. It is also common in traditional teaching to see teachers *modeling* mathematical discourse more than *eliciting* it from students (Hansen-Thomas, 2009). This has the effect of exposing students to the language of math, but it does not directly engage them in that language.

In contrast, teachers engaged in PBL or reform teaching engage with students in a different way. One case study (Celedón-Pattichis, 2011) found that an ESL teacher implementing a reform curriculum changed her questioning style over the 18 months of the study from using the IRE model to having students justify their ideas. Another study by Hansen-Thomas (2009) comparing three teachers of ELs found that the teacher who showed the most commitment to reform math also did by far the most eliciting of student discourse from her students. The ELs in her class also went on to grow from getting failing scores on standardized math tests to getting average or above-average scores, compared to the teacher who did the least eliciting of student discourse, whose ELs scored the lowest of the ELs in the study (Hansen-Thomas, 2009). From this, we can infer that promoting student discourse in a reform classroom has the potential to significantly benefit ELs in their understanding of mathematics.

Students in reform classrooms spend much more time speaking about math than those in traditional classrooms dominated by the IRE model. This is because working in collaborative problem solving groups means that students are talking to each other, rather than just the teacher. MacDonald (2017) claims this structure creates less pressure for ELs, making them more willing to share their thinking than in a public IRE interaction.

This is essential, because ELs engaged in all four modes of language - that is, speaking, reading, listening and writing - in a lesson learn the content and the language more completely (Echevarria et al., 2000). However, group work needs to be implemented with purpose; MacDonald (2017) cautions that without clear goals and roles, ELs can often be delegated the roles of simply passively listening or writing for the group, rather than engaging directly in the dialogue. If it is done well, however, ELs get a chance to hear and use mathematical terminology and receive coaching from their teachers in using more precise language (Hansen-Thomas, 2009), which MacDonald et al. (2014) claim also helps them learn more precise mathematical thinking.

In conclusion, while there is insufficient evidence to claim that PBL should be used to the exclusion of traditional teaching across all classrooms, it dramatically increases the amount of time ELs spend communicating about math, which is beneficial to both their language acquisition (specifically cognitive academic language proficiency) and their mathematical thinking. However, in order to engage with solving complex problems, ELs need the tools to understand these problems. The next sections will examine several word problem solving strategies discussed in literature, some of which may be beneficial to the problem solving process, and others of which have been discredited, beginning with the previously-discussed key word strategy.

The Key Word Strategy

According to Hegarty et al. (1995), students generally fall into one of two main approaches to solving word problems: making sense of the situation described in the word problem, or searching for numbers and key words in the text combine into an expression. They refer to the first approach the “problem-model” approach, and the second as the “direct translation” approach, although they note it goes by many names, including “key word”, “number grabbing”, and “compute first and think later”. Their research showed that successful problem solvers tend to use the problem-model approach, while unsuccessful problem solvers tend to employ direct translation.

Many other authors, including Boonen et al. (2016), Clement and Bernhard (2005), Dick et al. (2016), Van de Walle (2014), and Verschaffel et al. (1992), to name a few, have written about the insufficiency of the key word strategy. All of them essentially make the same point: the key word strategy is superficial and bypasses any attempt at *making sense* of the problem. A student who learns that *left* is a key word for subtraction and then can correctly execute a subtraction algorithm *at no point* engages with the *concept* of having a remaining quantity after the original quantity decreases. It is the experience of this researcher that this hypothetical student likely has no idea why his solution is correct, would be unable to justify it, and would struggle to replicate his success on a different problem without this key word.

According to Van de Walle (2014), this strategy is often introduced by teachers with good intentions of helping students understand the situations described in the problems. It seems that ELs are particularly targeted with this strategy; even researchers writing in the 2010s were still advocating teaching this method to ELs with this very intention (Chan, 2015; Orosco, 2014). After all, some word problems cannot be solved without understanding technical vocabulary, such as *sum* or *product*, as discussed earlier in this chapter. Reading further in these researchers’ papers, it is clear that both Chan (2015) and Orosco (2014) intended students to go deeper with their reasoning, rather than to simply follow a direct translation approach. However, Van de Walle (2014) goes on to say that, despite these more sophisticated intentions, “students often adopt the key-word approach as a replacement for making sense of the situations” (p. 122). On the other hand, it is the experience of this researcher that some teachers do, in fact, only teach this strategy in its superficial form, never engaging students on a deeper sense-making level.

There are many situations described in word problems that confound users of the key word strategy. One is that some key words are inherently ambiguous, such as “each”, which can imply either multiplication or division. Other problems might contain multiple key words that would individually imply different operations from each other (Van de Walle, 2014), such as “each” and “total”, which together might imply multiplication, but can mislead a number-grabbing student to add instead. Van de Walle also discusses further confounding problems, which have no key words at all (such as those described in Chapter One), or require multiple steps with different operations to solve, relying on a nuanced understanding of the relationships in the situation.

One type of problem, however, is so well known to baffle students employing the key word strategy that multiple studies (Boonen et al., 2016; Lewis & Mayer, 1987; Van der Schoot et al., 2009; Verschaffel et al., 1992) have examined how students approach them. These problems are termed “inconsistent language problems” by Lewis and Mayer (1987), and they are distinguished by presenting the information in a confusing order. The example given by Lewis and Mayer reads “there are six times as many students as professors at this university” (p.364), which often prompts students to multiply the number of *students* by six, rather than the number of *professors.*

These problems are known to be more difficult than “consistent language problems” for many students (Lewis & Mayer, 1987; Verschaffel et al., 1992). To make things more difficult still, many of these problems contain a misleading key word (Verschaffel et al., 1992). For example, consider the following word problem:

Ben has $5 less than Alex. Ben has $10. How much does Alex have?

This is an inconsistent language problem because Alex, the person with the unknown amount of money, is the *object* of the first sentence, rather than its subject. Adding to the confusion, the problem includes the key word “less”, which often implies subtraction. However, because Ben’s amount of money is already known, and it is the *greater* quantity that is missing, one must *add* 5 and 10 to arrive at the correct answer. Problems where the key word is a distractor lead to errors using the key word strategy (Boonen et al., 2016; Van de Walle, 2014; Verschaffel, et. al, 1992).

As this approach to solving problems appears to be inadequate, the next section of this chapter will examine strategies described in the literature that encourage students to make sense of the problems they are trying to solve.

Word Problem Solving Strategies

As word problem solving is a relatively complex and challenging skill (Boonen et al., 2016), a wide variety of strategies have emerged to assist students with performing it. The authors and researchers describing them typically target them at a particular audience, such as high school physics students (Heater et al., 2012) or third graders at risk of mathematics disabilities (Griffin & Jitendra, 2008), although they often provide thoughts about their applicability outside of the group discussed in the article. For example, in a guide meant for teacher professional development, Jitendra (2004) asserts that her Schema-Based Instruction (SBI) strategy is appropriate and effective for students from kindergarten through college, even though her study with Griffin (2008) only describes its use in a third grade classroom.

Although the strategies described are numerous, several patterns emerge. Firstly, the authors of some strategies, such as Driver and Powell (2017), target their strategies to students with Mathematics Difficulty and specify the importance of using explicit instruction with their strategies. Others, such as Dick et al. (2016), do not specify a targeted group, and contextualize their approach in reform, problem-based instruction, relying heavily on small problem-solving groups. Another pattern is the popular use of acronyms to help students recall the steps of a strategy, as used by Hohn and Frey (2002), among others. A third pattern is the high popularity of using diagramming among the strategies surveyed here.

Next, the final two parts of this section will discuss two broad approaches observed in different articles. Some strategies approach problem solving from the perspective of using the same techniques students use to read stories, such as stopping to make predictions. Lastly, the field of Schema-Based Instruction, or SBI, is discussed, as several authors, such as Griffin and Jitendra (2008) apply this approach to teaching mathematics in closely related ways.

Targeted Populations and Philosophical Approach. One way to differentiate between the strategies is the targeted population; some target the general population, while others target students with Mathematics Difficulties (MD). Driver and Powell (2017) describe students with MD as students whose math performance is low, with or without a diagnosed mathematics learning disability (dyscalculia). According to Boaler (2016), approximately 95% of students are capable of high achievement in any level of mathematics, describing the other 5% as those with math learning disabilities. It should be noted that Driver and Powell's (2017) definition of students with MD includes struggling students beyond Boaler's (2016) asserted 5% of students with learning disabilities.

The many strategies that specifically target students with MD tend to prescribe explicit instruction (Driver & Powell, 2017; Griffin & Jitendra, 2008; Kim, S. A., Wang, P., & Michaels, 2015; Orosco, 2014). These authors describe how explicit instruction follows the general pattern of having the teacher explain and model the strategy, then having students practice it individually or in small groups.

On the other end of the spectrum, several articles (Boaler, 2016; Dick et al., 2016; Nessel & Graham, 2006; Wiest, 2008) describe strategies for word problems that rely on reform-style instruction, in which the students' make sense of word problems by discussing them in small groups, as described earlier in this chapter. These authors often target the general population with their strategies, but Wiest (2008) specifically targets ELs. However, it is noteworthy that none of them is specifically targeting those students with MD.

As a mathematics teacher, this research supports the idea that the majority of students would be well-served by a reform-oriented strategy, but applying explicitly taught strategies may be appropriate as an intervention for students who appear to experience MD. However different the strategies may be in their philosophy of teaching, two patterns frequently appear in strategies on both sides of the divide: the use of acronyms and diagramming.

Acronyms. As mentioned in Chapter One, a common pattern among word problem strategies is the use of acronyms to help students remember the mental steps of problem-solving. Hohn and Frey (2002) describe these acronyms as heuristics that give the students a more systematic way to solve problems. An example of a problem-solving heuristic is the SKATE strategy described in Chapter One, in which students would (1) Survey or skim the problem to understand the big picture, (2) Keep or write down the important information, (3) Attempt to estimate, (4) Take time to solve the problem and (5) Examine their answers to make sure they made sense. Several acronyms from the literature, representing the work of authors from across the explicit-reform spectrum, are listed below:

- PIES: Picture, Information, Equation, Solve (Heater et al., 2012).
- RAT Math: Read-And-Think Math (Nessel & Graham, 2006).
- RISE: Read the problem, Illustrate the problem, Solve for the unknown information, and Explain what you solved for (Driver & Powell, 2017).
- SOLVED: State the problem, Options to use, Links to the past, Visual aid, Execute your answer, and Do check back (Hohn & Frey, 2002).

These authors often describe how some students particularly benefit from the use of acronyms. Heater et al. (2012) assert that acronyms are particularly helpful for students with learning disabilities. Hohn and Frey (2002), on the other hand, identify students who struggle with math and girls as two groups who may benefit from problem solving heuristics more than others. In particular, they note that girls tend to rely on visual aids provided by their teachers more, while boys tend to rely on strategies and procedures they came up with themselves. However, the results of their study around the use of the SOLVED strategy did not directly support that speculation.

While all these strategies have different names and acronyms, most of them embed very similar steps. Most begin with a step of reading the problem or putting into one's own words. Students then draw a picture or diagram, perform the necessary calculations, then reflect on their answer in some way. Most of these acronyms describe steps to be performed in order, although RAT Math (Nessel & Graham, 2006) is a notable exception, in that students are meant to apply the procedure of reading a sentence and then thinking/making predictions about the problem repeatedly while solving a single problem.

It is unclear from the literature whether any particular acronym is preferable over others; what appears to be important is that they provide an easy-to-remember procedure that encodes the mental habits of problem solving, which may be helpful to struggling students. Another common element of word problem strategies that transcends the explicit-reform dichotomy is students diagramming a problem before solving it.

Problem Solving Using Diagramming. The use of pictures or diagrams appears in the vast majority of strategies examined in this chapter (Banerjee, 2010; Clement & Bernhard, 2005; Driver & Powell, 2017; Griffin & Jitendra, 2008; Heater et al., 2012; Hohn & Frey, 2002). Like the use of acronyms, this element of problem solving is so ubiquitous that it is not confined to either strategies aligned with explicit or reform instruction. However, after digging deeper, it becomes apparent that the pictures and diagrams serve different purposes in different strategies. In the PIES (Picture, Information, Equation, Solve) strategy, the picture's purpose is simply to get the student started with making sense of the problem, as opposed to simply giving up and guessing (Heater et al., 2012). Later on, the drawing is labeled with the important information in the problem, as shown below (Figure 2):

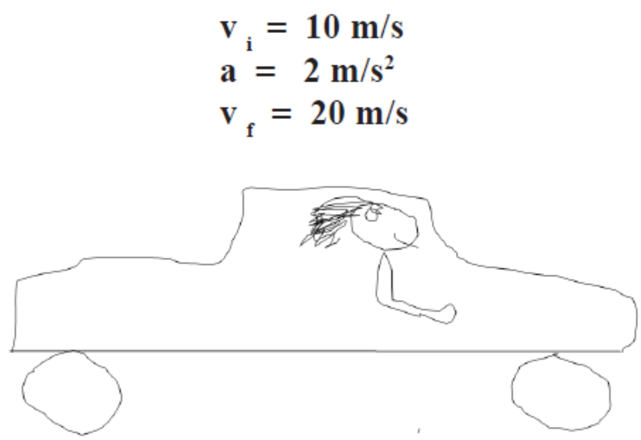


Figure 2. A diagram being used with the PIES strategy. Used with permission. (Heater et al., 2012)

Griffin and Jitendra (2008), on the other hand, use carefully pre-generated diagrams that align with different types of problems. In their SBI (Schema-Based Instruction) strategy, students with Mathematics Difficulty (MD) are explicitly taught these diagrams to help them make sense of different categories of word problems. Students sort the information in the problem into the different spaces in the diagram to help them organize their thoughts when choosing an appropriate operation. This strategy will be discussed in more detail later, but an example of one of Jitendra's diagrams is provided below. This diagram would be used to organize information in a problem in which one quantity (such as an amount of money, or a person's age) changes over time (Figure 3).

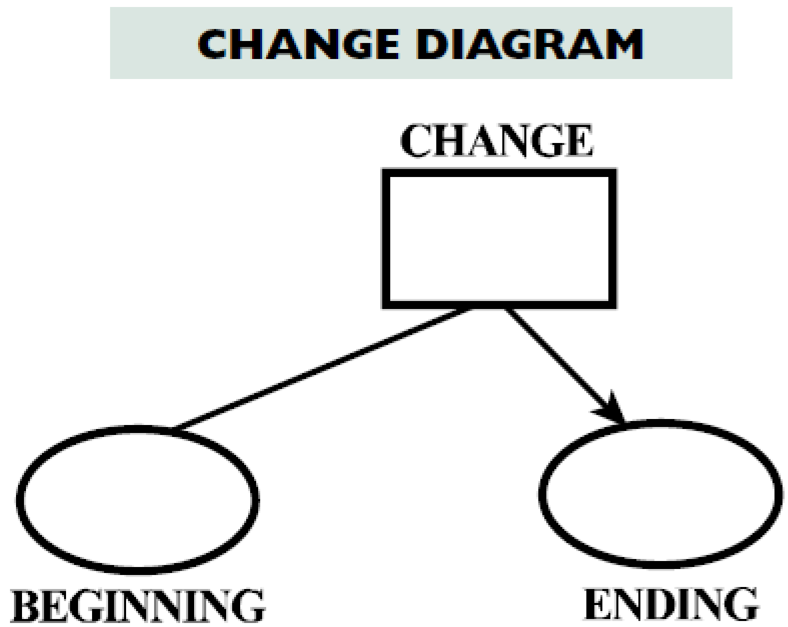


Figure 3. A diagram for solving a change problem in SBI. Used with permission. (Jitendra, 2004)

A third use of diagramming, more common in reform-oriented strategies, is precisely showing the mathematical relationship described in the text. Clement and Bernhard (2005) demonstrate this with bar diagrams of fractions, labeled with contextual information from the problem. Like with SBI, the intent is to help students make sense of the problem. The key difference here is that the intent of the diagram is giving a visual representation of the math, rather than organizing information to aid in selecting an operation to perform. The diagrams described in this article, as well as in the CPM curriculum, could be used as an integral tool in *performing* the math itself. For example, below is a diagram provided by Clement and Bernhard for finding how much weight someone lost after starting a diet (Figure 4):

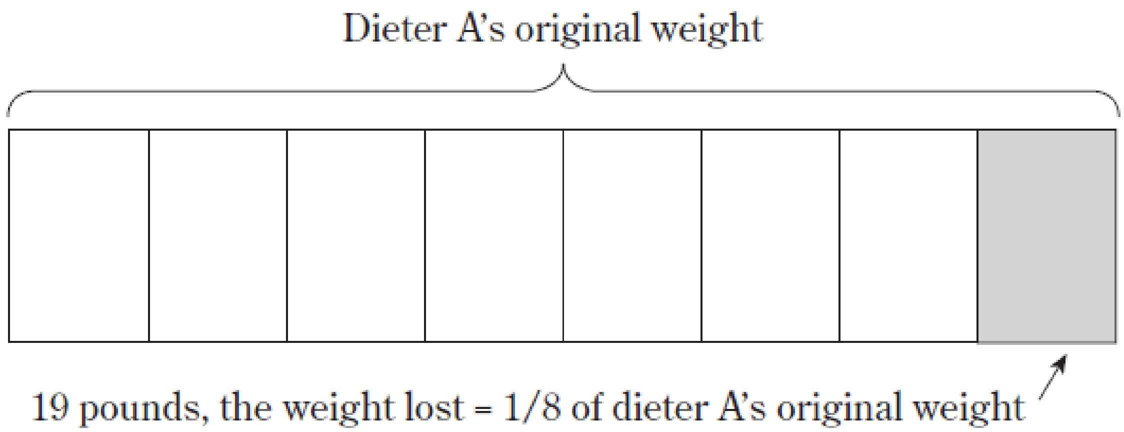


Figure 4. A diagram used in a reform approach. Used with permission. (Clement & Bernhard, 2005)

While the use of acronyms and diagramming are both used across the philosophical spectrum, the remainder of this section will examine two narrower themes among problem solving strategies described in the literature.

Story-Oriented Strategies. Two strategies found in the literature - Mathematical Bet Lines (Dick et al., 2016), from an NCTM publication, indicating a reform orientation, and RAT Math (Nessel & Graham, 2006) - are so similar they could be considered essentially the same strategy. In both, the problem solving process is turned into a conversation between the teacher and the class. The key is that this strategy feeds students the information in the problem line-by-line and asks them to make predictions (or "bets") about what the question will be. For example, a teacher might start by giving the following information:

James earns $8.24 per hour at his job.

At this point, referred to as a "bet line" by Dick et al. (2016), the teacher would pause to ask the class to predict what information will be given next. Nessel and Graham

(2006) specify that the students should spend a few minutes discussing this in small groups before bringing the whole class back together. Some students might guess that they will learn how many hours James worked. Others might guess that James wants to buy something that costs $100. After getting two to three predictions from the class, the teacher continues:

This week, James worked 20.5 hours.

Again, the teacher pauses and asks for predictions, but this time for what the *question* will be. Some predictions will flow directly from the information given, such as “how much did James earn in total?” Others may be more inventive, such as “how much more did James earn this week than last week?, ” to which the teacher might react by saying “That sounds like a difficult problem to solve!” This process, regardless of what it is called, puts the emphasis on understanding the story of the word problem, rather than on getting an answer (Dick et al., 2016). Nessel and Graham (2006) add that using this approach means students may end up generating very interesting questions along the way, which the class can then solve.

Both versions of this approach are meant to be used with the whole class, so that a diverse array of students will benefit from verbalizing and hearing other students verbalize their thought process. In contrast, the last approach examined in this chapter is meant to be explicitly taught to small groups of struggling students.

Schema-Based Instruction. One field of study that turned up in numerous articles (Driver & Powell, 2017; Griffin & Jitendra, 2008; Mitsugi, 2017) is referred to as Schema-Based Instruction, or SBI. SBI is one of the teacher-centered strategies targeting students with MD (Griffin & Jitendra, 2008), but its insights are potentially applicable to other students as well. For example, students using the key word strategy have not yet learned to use reasoning to identify what type of problem they are solving, so they could potentially benefit from this approach. ELs who have not developed a schema for these different problem types could also benefit. The key idea of SBI is that the situations in word problems tend to fall into several categories related to the different operations.

For example, Griffin and Jitendra (2008) identify the three types of addition/ subtraction problems as “change,” “group,” and “compare.” Each of these problem types might imply addition or subtraction depending on what information is known, and what information must be found by the problem solver. Below is the example of a compare problem given in the section on the Key Word Strategy:

Ben has $5 less than Alex. Ben has $10. How much does Alex have?

Here the information given includes Ben’s money and the difference between the two people’s money. The unknown information is Alex’s money, which we know is the greater quantity. Therefore, addition must be used to solve it (despite the misleading use of the word “less”).

A teacher employing SBI would teach the students how to draw a diagram for a compare problem, with spaces for the two quantities being compared and a space for the difference. Students would solve this problem by first identifying that this is a compare problem (implied by the first sentence), draw the appropriate diagram, write $10 in Ben’s space, write $5 in the space for the difference, and notice that Alex’s space is the empty one. The student could then see that the greater of the two numbers is the unknown one, and therefore it could be solved using addition.

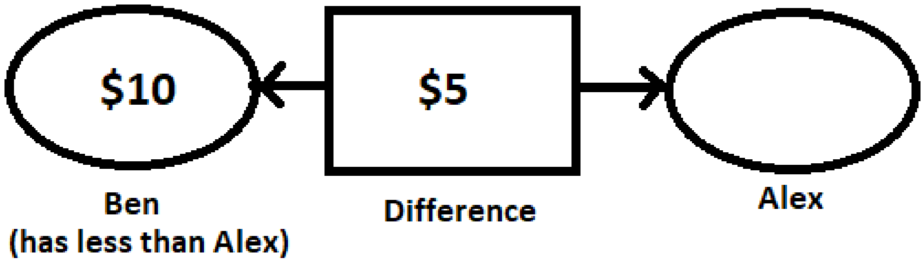


Figure 5. A diagram that could be used to solve a compare problem. Created by the author, based on a diagram by Jitendra, used with permission (2004)

SBI goes by a variety of names. Driver and Powell (2017) list some alternatives: Schema Instruction, and Schema-Broadening Instruction. The name SBI and its underlying theory - that is, explicitly teaching students the core schema behind an idea - is applied in areas beyond mathematics too (Mitsugi, 2017). This notion of teaching students to identify the problem type is also referenced in Hohn and Frey's (2002) SOLVED strategy, described in the section on acronyms.

The work of all the researchers described in this chapter is hardly an exhaustive list of all the word problem strategies being used today. However, there are ample approaches to choose from, whether guiding the entire class in the habits of effective problem solvers or performing an intervention with a small group of students with MD. Regardless of the particularities, each strategy could be used to encourage students to adopt a problem-model approach to word problem solving.

CHAPTER THREE

Methodology

Overview of the Chapter

The purpose of this study comes in two phases. The first was to identify different teaching strategies that support 7th graders with intermediate to advanced levels of English language proficiency to interrogate mathematics word problems. The second was to observe how these 7th graders respond to said strategies. My goal was to implement a word problem solving strategy that incorporates the Read-And-Think (RAT) method and diagramming to scaffold students' problem solving process and, hopefully, make them more effective learners in a Problem-Based-Learning (PBL) classroom.

The study explored the question: *how do students with varying levels of English proficiency respond to identified teaching strategies noted in the research literature that support them with developing a problem-model approach to solving mathematics word problems?* A "problem-model" approach refers to when a student uses the text to make sense of the mathematical situation in a word problem. For the purposes of this study, the Read-and-Think strategy was used to attempt to support students in developing a problem-model approach. This was in comparison to students attempting to translate verbal sentences directly into equations or expressions (the "direct-translation" approach).

In this chapter, I will justify my use of a qualitative research paradigm to address the capstone question. The specific research design used is an action research methodology (Mills, 2017). I will also elaborate on my setting, participants, procedures, and data collection. Finally, I will describe how I analyzed my data.

Qualitative Research Paradigm

According to Creswell and Creswell (2018, p. 16), qualitative research seeks to answer open-ended questions by interpreting "the themes or patterns that emerge from the data." I am investigating a question of how students respond to word problem strategy instruction, which is an open-ended question. My methods also involved examining student work and verbal explanations in order to analyze their process, rather than simply quantifying how many questions they got correct. This is inherently a process of searching for patterns that emerge in the data. The problem solving process is creative in nature, and so qualitative tools seem the most appropriate for understanding this process and how it does or does not change.

Some of the qualitative data taken for this study was quantified in the form of scoring word problems for correctness according to a rubric. However, I used a small convenience sample for the study, consisting of the students in my own classroom whose families were willing to have their data used for this study. Therefore this would be insufficient to use the statistical analysis methods Creswell and Creswell (2018) associate with quantitative research.

Action Research Methods

This study falls under the category of Action Research, which Mills (2011) defined broadly as "any systematic inquiry conducted by teacher researchers, principals, school counselors, or other stakeholders in the teacher/learning environment to gather information about how their particular schools operate, how they teach, and how well their students learn" (p. 5). This study was accordingly conducted by a teacher in his classroom to gain insight into how students respond in their learning after a teaching intervention (R.A.T. Math) takes place. Mills also emphasizes that Action Research serves to drive educational change within the setting it is conducted in, which was also my aim. Should this intervention prove successful, I intend to teach other secondary mathematics teachers in my school to apply it in their own classrooms. Should it prove unsuccessful, I will continue to reflect and seek out other interventions.

Mills (2011) also lays out four basic stages to the Action Research process: first, identifying an "area of focus"; second, collecting data; third, analyzing and interpreting the data; and, fourth, developing "an action plan" (p. 5). This process is also cyclic in nature; drawing conclusions and developing an action plan is meant to raise new questions and spark a new wave of collecting data. The area of focus has been thoroughly described in Chapter One. The data collection and analysis techniques are described below. The action plan will be described in Chapter Five, which involves sharing my findings with fellow secondary math teachers, instructional coaches and administrators, and possibly training them in techniques that appear to be effective.

Location/Setting

The setting of this study is a large urban k-12 public charter school focusing on southeast Asian diaspora populations. The school is one of the largest in the city, with a student population of over 2200, and is located in a large metro area of a major city in the upper midwest. In Academic Year (AY) 2017-18, 42.3% of middle schoolers were proficient in math and 34.9% were proficient in reading according to state standardized testing data. Thirty-one percent of middle schoolers were classified as English Learners (ELs), 10% as Special Education students, and 81% qualified for free or reduced price lunches. ELs are either given separate classes taught by the ESL staff or mainstreamed into general education classes depending on their English Language Proficiency (ELP) level.

At this school, I teach three sections of 7th grade math. All three are considered “regular”; most students in special education, those taking honors math, and those whose ELP level does not qualify them for mainstream math classes are all taught by other teachers. Classes have anywhere from 20 to 31 students aged 12 to 13, few of whom have IEPs, as those students are clustered into a co-taught class, which is taught by a different 7th grade math teacher. 36% of the students in these classes are classified as ELs. On WIDA’s assessments (the organization that develops testing for ELs’ language proficiency), their language levels range from 1.9, indicating “beginning” level of ELP, to a 4.5, indicating an “expanding” level of ELP, which is also the minimum score to be exited from ESL services. The mean score is a 3.5, indicating a “developing” level of proficiency.

In AY 2018-19, the school adopted the College Preparatory Mathematics (CPM) (CPM Educational Program, 2013) curriculum for all students in grades 6-12. This provided much of the impetus for selecting problem solving for ELs as the topic for this study. As the collection of data took place in AY 2019-20, one implication of this is that most participants in the study had at least one year of learning math through PBL prior to beginning this study. Conducting a study like this with students who had no experience with PBL could lead to different results. Finally, this school provides every secondary student with a Chromebook for use in their classes (a one-to-one technology environment).

Participants

I recruited 21 of the 78 students from my own classroom with varying levels (intermediate to advanced) of ELP. These were all 7th graders (aged 12-13) in mainstream math classrooms. 7 participants were officially designated as ELs in the school's records, but none had WIDA scores below a 2, which would indicate a beginner's proficiency with the English language, as these students are typically placed in separate math classes. 2 participants were exited from the ESL program, as their overall WIDA scores exceeded 4.5 with none of the four domains (speaking, listening, reading, and writing) scoring below a 3.5. Some participants still may have never been identified as ELs when they first enrolled if that was before the school's ESL policies were updated. I could not identify these students with certainty, but the four students who spoke another language at home and did not meet the state reading standards on standardized tests are treated as their own category, called "borderline" for the sake of convenience. For the sake of getting a complete picture of the students in my classes, I also examined the work of 6 students who speak another language at home but at either partially or fully met the state reading standards on standardized tests. Finally, I also examined the work of 2 students who speak English at home and either partially or fully met the state reading standards on standardized tests.

I used NWEA/MAP scores (a test commonly given to track student growth in reading and math over time) to help me identify these last two categories of students for the study; students who are within one standard deviation of the mean reading score for 7th graders at the start of the year will be considered "proficient" in English for the purposes of this study. I also compared this with their scores on the state standardized tests for reading. The school determines language spoken at home using a Home Language Questionnaire, and the school agreed to give me access to this data. All students in my classroom completed the assignments being examined for this study, but only these 21 students' work for whom consent was obtained were used for the study.

Data Collection

I gathered data from four sources, before, during, and after the study takes place. The purpose of using the four data collection methods is to triangulate the data, and to provide four different ways to glimpse the students' problem solving process. According to Mills (2011), "the strength of qualitative research lies in its triangulation, collecting information in many ways, rather than relying solely on one" (p.56). These four sources were a student word problem warm up, a verbal account of the problem solving process recorded with Flipgrid (an online video response tool), the students' grades and problem solving work on individual tests, and a survey in which students indicated their confidence level in having a strategy to solve word problems and the in explaining their thinking to others. I also kept a field journal over the course of the study in order to keep track of what lessons were being taught on which days, instructional moves I intended to make in order to promote a problem-model approach, instructional changes or decisions I made while teaching the class, and my observations around changes or breakthroughs in student thinking that become evident in class or group discussions. For the benefit of the reader, a chart is provided below to clarify these tools and when they were administered:

Table 1

Pacing of Data Collection Tools

	Warm Up and Survey	Flipgrid Video	Individual Test	Field Journal
Prior to Study (5 weeks)	Introduced to class during routine-building phase.		Not given	Not kept
Pre-Assessment (Week 1 of study, Week 6 of school year)	Collected Week 1, Day 3	Collected Week 1, Day 3	Collected Week 2, Day 1	
Mid-Point Assessment (Week 6 of Study, Week 11 of school year)	Collected Week 6, Day 1	Collected Week 6, Day 1	Collected Week 6, Day 2	Maintained continuously
Post-Assessment (Week 10 of Study, Week 15 of school year)	Collected Week 10, Day 5	Collected Week 10, Day 5	Collected Week 10, Day 3	

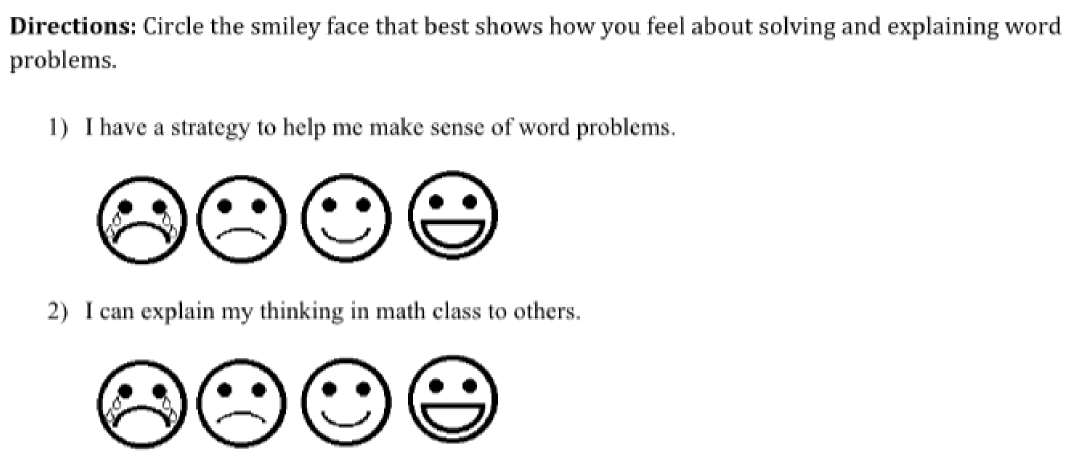
An added benefit of using four data sources is that each one engages the student with different modalities. For example, the warm up engages students in, at least, reading, listening and writing, while the Flipgrid video highlights the students' ability to speak about math. They also all give insight into different elements of the students' thinking and attitudes.

Students in my classroom routinely complete a short warm up during the first 5-10 minutes of class. They are allowed to collaborate on these, in keeping with PBL and reform math ideas about collaboration, group work, and communication, as described in Chapter Two. These are used as a formative assessment tool, a chance for students to activate prior knowledge, as well as a way for students to get individual written feedback on their work on a daily basis. I planned three particular warm ups, one at the beginning, one at the middle, and one at the end of the study to contain a single, multiple-step word problem for the students to solve, as seen in Appendix D. Mills (2011) defines this data collection tool as a student artifact, which can provide insight into a students' process.

The calculations required for the word problem on the warm up were limited to the four basic operations (addition, subtraction, multiplication, and division) and to numbers they can enter on a simple pocket calculator (positive whole numbers and decimals). The purpose of this is to try to limit the effect of students being unfamiliar with the mathematics required to solve the problem. They also implicitly drew on mathematical concepts such as ratios and areas, which students are generally expected to be proficient in by 7th grade.

The second data source was a verbal account of the problem solving process submitted through Flipgrid. In a Flipgrid response, students use the camera on their Chromebook to record a short (less than one minute and 30 seconds) response to a question. The tool allows students to record their response multiple times, until they are satisfied with the version they recorded. Many teachers at the school have noticed that the ELs they work with appear to prefer answering questions through Flipgrid than in a traditional format of verbally answering a question in front of their peers, perhaps in part due to the chance they have to rehearse their answer, which boosts their confidence. This format would be more efficient and less invasive than a traditional interview for two reasons. Firstly, it is more efficient because all students will be able to simultaneously explain their methods. Secondly, it is less invasive because the researcher is not sitting in and listening as the student speaks, hopefully limiting how much the researcher influences the student's response.

Thirdly, on the back of the warm up, the students were given a short Likert Scale survey. The students were asked to circle an emoji reflecting their feelings in response to two questions, representing a continuum from very negative to very positive. The survey I used is shown below:



Directions: Circle the smiley face that best shows how you feel about solving and explaining word problems.

1) I have a strategy to help me make sense of word problems.

2) I can explain my thinking in math class to others.

Figure 6. Survey for gauging student feelings about word problem solving. Created by the author.

Lastly, every unit or two, all my students complete an individual test of 10-12 questions. Two example questions are provided below. In keeping with recommendations from CPM (CPM Educational Program, 2013), the questions are cumulative in nature, with at least 50% of the material on the test being review questions. Most questions were presented in a short sentence format, although a few were more procedural in nature, simply asking students to perform an algorithm, such as one for dividing two fractions.

The data collected using this assessment was used to examine whether a student's individual performance has improved in the class, even when they are not allowed to collaborate with peers. Below are two questions from the CPM question bank, the first of which is procedural in nature, the second of which can be solved with several methods:

- *Show how to multiply (34)(51) using a generic rectangle. Fill in all dimensions and areas on the rectangle. What is the product?*
- *Needing some extra money, Bella cracked open her piggy bank. Out spilled a pile of pennies. When Bella arranged her pennies, she had eight groups of ten pennies with three left over. How many pennies does Bella have? Show your work to justify your answer.*

  (CPM Educational Program, 2013)

The tests had a mixture of these two types of questions, which allowed me to compare students' procedural fluency with their apparent conceptual understanding. In order to confound the direct translation approach, I made sure the word problems did not contain the most common key words. I also used the first question on each test to examine the students' problem solving approach when forced to work individually, instead of with their teammates.

Procedural Steps

In September of 2019, prior to beginning the study, I spent the first five weeks teaching my first unit and a half of the class, in which students were familiarized with classroom routines and the tools they would be interacting with as I took my data, particularly Flipgrid. Before doing my first wave of data collection, I had students complete two Flipgrid videos during class, which proved to be a very necessary step, as many students failed to complete and upload their videos within the time constraints they had to work with. I believe this time of learning classroom routines was important to ensure that the data was not affected simply by students becoming more familiar with the data collection tools. During this time, I also obtained informed consent from the students' guardians and the students themselves through written forms, available in both English and the students' home language, as well as gathering and interpreting my NWEA/MAP and state standardized test data.

Two weeks after Unit 1 was taught, I gathered baseline data for the students using the four tools described above. This is in keeping with CPM's recommendations for administering individual tests, as students need time to develop proficiency in the skills taught in Unit 1 through mixed, spaced practice (CPM Educational Program, 2013). When having the students complete the Flipgrid response, sentence frames were posted on the wall to help ELs come up with a response, and also emphasize that they need to provide *reasons* for their steps. For example: *First I* ______________________ *because* ______________________. I also had students complete the survey on the back of the warm up shown in Figure 1. I also noted the whole class's responses to this survey informally in my field journal to flesh out my impressions of the students' thinking and progress.

Once this baseline data was gathered, I taught one lesson outside of the normal flow of the CPM curriculum specifically about word problem solving. This followed the Mathematical Bet Lines/Read-And-Think (RAT) model described in Chapter Two. I projected a word problem onto the board, only showing the first sentence. We read the word problem together, stopping at each pre-selected "bet line" (the point at which students would “bet” what information would be given next) to discuss what we already knew, drew or continued drawing a diagram of the problem, and predicted what information we would find out next. Once all the information had been divulged, I asked the students for predictions about what the question would be. Finally, I revealed the question and had students attempt to solve the problem, then share their methods with each other.

If different groups have conflicting answers, we had classroom discussions around which methods make sense, led by the students as much as possible. At no point did I tell the students the correct answer, but I asked probing questions to guide their thinking. If the class could not work out which answer is correct, we moved on intending to return to discussing that problem later, once they had the tools to decide for themselves.

The rest of the lesson proceeded in much the same way, except gradually releasing more and more of the responsibility of drawing the diagram and interpreting the information in the word problem to the students. After this lesson, the RAT method was displayed in the room as an anchor chart, which I referred to when I found a team that appeared stuck with their problem solving process in normal classroom lessons.

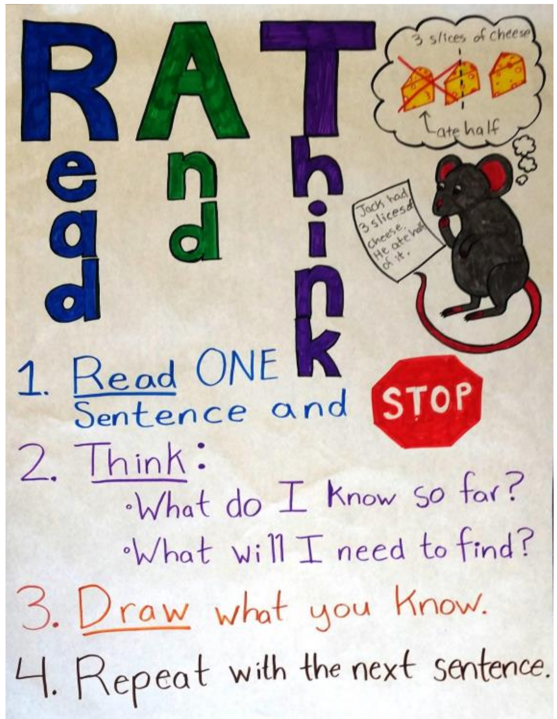


Figure 7. Read-And-Think anchor chart. Created by the author.

Six weeks (just past halfway) into the study, I administered my mid-way assessment, again using a word problem warm up, a Flipgrid response, the same exact survey, and an individual test. The purpose was to formatively assess whether students are recalling and applying the thinking of RAT Math in their own problem solving. I then taught a second RAT Math lesson after this was gathered to refresh students' memories about using this method and remind them that I expected them to use it when solving word problems. Finally, after ten weeks had passed, I administered the assessment tools one last time for analysis.

Materials

The materials needed for this study are already in routine use in classrooms throughout this school. One is having a Chromebook for each student so they may access the CPM eBook and the Flipgrid software. This is provided by the school, including copies of the eBook for each student. Another is the anchor chart described in the procedural steps section (see Figure 7). Finally, the CPM test-building software is required, in order to access the question bank for building the tests.

Data Analysis

As mentioned above, the purpose of this data is to glimpse the students' problem solving process and search for evidence of either using a problem-model approach, a direct translation approach, a mix of the two, or neither. Examples of students using neither include students who simply do not attempt to solve the problem at all, or students who randomly guess an answer.

When examining the word problem warm ups, I examined the students' written work for evidence of their process. To help distinguish problem-model approaches from direct-translation approaches, I specifically included misleading key words. For example:

James is making pancakes. He knows that 120 ml of milk will make 8 pancakes, and he notices he has 400 ml of milk left in the jug. How many pancakes will he be able to make?

In this word problem, a student following a direct translation model might be tempted to subtract some of the numbers due to the misleading use of "left" in the second sentence. Therefore, inappropriately subtracting would be taken as evidence of using a direct translation model.

Any attempt at drawing a picture that contains valid information from the problem, repeated addition of 120 + 120 and 8 + 8, setting up a proportion, or making a table could be taken as evidence of a problem-model approach. A valid expression or series of expressions showing 400 divided by 120 times 8 would also be taken as evidence of a problem-model approach, as it is unlikely a student following a direct-translation approach would arrive at these steps. Note that this is far from all the valid approaches to this problem - it is necessary to be open to unexpected methods, so long as they are consistent with the relationships involved. Also, a student truly making sense of the problem must make sense of the remainder (strictly speaking, either 40 ml of milk will be left over if the students insists on only making whole batches, or 10 ml will be left over if the student notes the unit rate of 15 ml of milk per pancake, or the student might realize that the remaining milk could be used to make a smaller pancake). *Any* attempt to make sense of the remainder can be taken as evidence of using a problem-model approach.

An incorrect answer showing no work might be a more-or-less random guess, but it is very likely that the students' thinking is present, just not expressed in writing. I made every effort to infer the method the student may have taken to arrive at the answer and identify whatever partial understandings the student may have had. For example, an answer of 3.33 would appear to come from a student dividing 400 by 120, which shows a partial, but incomplete understanding. Showing one's work is already an expectation in my classroom, and I emphasized this as students worked on the warm up, but I was prepared to have some hard-to-interpret data like this.

When analyzing the Flipgrid responses, I listened for evidence of the student making sense of the problem (for example, referring to or rephrasing details in the problem, showing pictures or diagrams they drew, justifying steps instead of just listing them). I wrote a rough transcription of each response for the 21 students in the study, noting the language they used to describe their process (see Appendix H). I anticipated that some students might say "I just guessed" or "I don't know", which might mean that they are not following either problem solving model, or that they simply have not developed the language or confidence to describe their process. Therefore, it was important to compare their response on Flipgrid with their written work to try to tell the difference. Some students may also admit to copying off of a neighbor, as sometimes in my experience students can be surprisingly honest. While collaboration with neighbors is encouraged on these warm ups, it is expected in a PBL classroom that students be able to explain a solution they got from a teammate, not just have it written down on paper.

Students who described looking at specific words and then described inappropriate operations are most likely following a direct translation approach (a response that I anticipated but, interestingly, did not come across). On the other hand, students providing justifications for their steps, citing parts of the situation described in the problem rather than specific words were likely using a problem-model approach, but simply misunderstanding the situation. Finally, some students simply listed a series of steps, sometimes fixating on the procedural details. For example, a student might go into intense detail about every step of performing long division in the problem. It was ambiguous what model these students are following, and their response needed to be compared with their written work to obtain a more complete understanding of their thinking. However, a student explaining this way is unlikely to be completely following the problem-model approach, and often explanations like this stopped making sense when they became focused purely on procedures.

Also, following Mills' (2011) recommendation, I listened for patterns in students' Flipgrid responses in order to identify recurring themes, including ones I did not expect or was explicitly searching for.

Finally, the individual tests were graded according to the CPM rubric. This rubric is shown below:

4-Point Rubric
For Scoring a Single Problem or Task

4 Fully Accomplishes the Purpose of the Task

Student work shows full grasp and use of the central mathematical idea(s).

Recorded work communicates thinking clearly using some combination of written, symbolic, or visual means.

3 Substantially Accomplishes the Purpose of the Task

Student work shows essential grasp of the central mathematical idea(s).

Recorded work in large part communicates the thinking.

2 Partially Accomplishes the Purpose of the Task

Student work shows partial but limited grasp of the central mathematical idea(s).

Recorded work may be incomplete, somewhat misdirected, or not clearly presented.

1 Makes Little or No Progress Toward Accomplishing the Task

Shows little or no grasp of the central mathematical idea(s).

Recorded work is barely (if at all) comprehensible.

0 Makes No Effort

Figure 8. A problem solving rubric from CPM. Used with permission. (CPM Educational Program, 2013)

I prepared scoring guides for each question to anticipate what this looks like for each question. For example, a student asked to find the area of a triangle described in a word problem would receive a 4 for completing an accurate diagram and/or a clear number expression showing their thinking and a correct answer, correctly labeled with appropriate units (such as square inches). A student would receive a 3 for work following the same process as the student who earned a 4, except there might be minor scratchwork errors or missing/inappropriately labeled units. A student would receive a 2 if they completed part of the process correctly, but not the whole thing: for example, this is the grade a student would receive for correctly drawing and labeling the dimensions of the triangle, but failing to divide their answer by two. A student would receive a 1 for following any inappropriate method, such as drawing the wrong shape, incorrectly labeling parts of it, attempting to find the perimeter instead of the area, or other such mistakes that miss the point of the problem. Finally, a blank question receives a zero.

When interpreting the test, I simultaneously used their test score as a piece of quantitative data and more closely examining one question. The question I examined was one that uses inconsistent language or contains no key words. The purpose of examining this one question was to see how the students approach problem solving when they are not allowed to collaborate.

I also used the field journal at this stage to give context to the other 3 data sources. This will help me make sense of any changes to the students' problem solving approach observed in the data, especially if there are discrepancies between data sources.

Another data analysis technique Mills (2011) recommends is to state what is missing from the data. I will state it in Chapter 4 in order to not make misleading claims, and also to identify further data collection tools I may want to bring in to gain a fuller understanding of how my students are approaching problem solving.

Ethics

As this study worked with human subjects, specifically minors, many of whom can be considered educationally disadvantaged (some were ELs, refugees, and/or living in poverty), ethical procedures were followed to ensure their well-being. I followed the procedures laid out by Hamline University's Internal Review Board.

All students participating in the study were given voluntary informed consent forms in their own language. They were asked to bring these forms home to their parents to be signed. Parents had the option to opt out of the study and not have their students' data be used. Out of respect for the individual students, students themselves also had the option to opt out of the study. Thus a parent could have given permission for their child to participate but the student could have decided not to. This choice by the students was respected. These consent forms also laid out potential benefits (helping their math teacher to understand how to better instruct their child in math) and risks (potential for a data breach in which students' identities are revealed).

In order to protect students' identities, the data collected was de-identified as quickly as possible. Real names on warm-ups and unit tests were replaced with pseudonyms and the list linking the real students' identities with the pseudonyms was kept separately from the data in a locked location. The data file itself was password protected and saved in a separate location under a separate Google Drive account with a different password from the participant list. All Flipgrid summaries were also only labeled with student pseudonyms. All materials collected for the study will be destroyed at the conclusion of Academic Year 2019-2020.

Limitations of the Research Design

The nature of this study limits how the results may be interpreted. As the research design is qualitative in nature and uses a small convenience sample, the results cannot be generalized to all ELs. As Mills (2011) explains, it is more important that Action Research fully describes its setting (see Location/Setting above) so that others working in a similar setting might see how the study's findings might apply to them, rather than assume that the findings apply to settings that are dissimilar from the one in which the study was conducted. However, the study's findings may prove transferable to other math classrooms with high EL populations using reform curricula.

As this study uses an Action Research methodology, I could influence the results simply by trying something new in the classroom. It is also unclear whether a 10-week time frame is sufficient for all students to develop a problem-model approach to problem solving, or whether they will continue this approach after the end of the study. Finally, due to my position as both the researcher and the participants' teacher, there may be some inherent bias to my data analysis, which I have addressed in the Action Research Methods section above.

In order to increase the trustworthiness of my methods and results I took the following measures. Using Guba's criteria for validity of qualitative research, as outlined by Mills (2011), I will take measures to address my study's credibility, dependability, and confirmability. Credibility, or "the researcher's ability to take into account the complexities that present themselves in a study and to deal with patterns that are not easily explained" (Mills, 2011, p.85) was addressed by spending the first unit of the school year getting students accustomed to the routines and tools being used in the study before beginning data collection, and then triangulating the data, as described above, and testing my findings using all four data sources. I addressed this study's dependability as well by triangulating the data, and also making my data available to others to verify my findings for themselves (Mills, 2011).

This study's confirmability, or "neutrality or objectivity of the data that has been collected" (Mills, 2011, p.86) was addressed again by triangulation, and by recording my own assumptions or biases beforehand. Namely, my main assumptions are my expectation that the RAT Math strategy will have a beneficial impact on students' word problem solving and my desire to attribute failure to see improvement will to students' reluctance to actually practice the strategy with fidelity. Having named these biases, I strove to find evidence to the contrary and seek other explanations should the RAT math strategy prove ineffective within the course of this study.

It should also be noted when examining the data that due to a number of factors, including what skills had been taught at the beginning of the year and that all 7th grade math teachers at the school were expected to collaborate on test creation, the first individual test included a higher number of procedural questions compared to the other two. This may have influenced how student achievement changed throughout the study.

Conclusion

In this chapter, I laid out my justification for using a qualitative research paradigm and an action research methodology. I described the setting of my study, the students I hope will participate, the data collection tools I will use, and how I will analyze the data gathered through them. In Chapter Four I will describe the results of this analysis.

CHAPTER FOUR

Results

This chapter presents the results of all the data gathered for the study: the warm-up problems, the corresponding Flipgrid videos, the attitude surveys and the individual test questions. This data helped answer the question: *how do students with varying levels of English proficiency respond to identified teaching strategies noted in the research literature that support them with developing a "problem-model approach" to solving mathematics word problems?* This chapter will also, therefore, discuss how students' approaches to problem solving changed or did not change over the course of the study. It will also discuss patterns that emerged from the data when viewed through the lenses of English Language Proficiency (ELP), initial approach to problem solving, and explanation style.

Students' Background Data

My original aim was to focus this study on 10 students of varying ELP levels. I well exceeded my initial goal, attracting 21 participants. Because the school seems to have a large number of students who might have been classified as ELs under different circumstances but who, for one reason or another, are not, I split the participants into five groups: ELs as identified in school records, Exited ELs (those who once received services but for whom they had been discontinued), the students I have labeled "borderline": those who speak Hmong at home and struggle with reading (as identified by standardized testing data), students who speak Hmong at home and who scored within one standard deviation of the mean on the NWEA/MAP reading test, and students who speak English at home. It should be noted that one participant (Student C) is Karen, another (Student E) is African American, and the rest are Hmong. The students are listed below, according to their categorization:

Student	Category	Math Proficiency	Reading Proficiency	WIDA Score
Student A	EL	Meets	Meets	4.5
Student B	EL	Does Not Meet	Does Not Meet	3.6
Student C	EL	Does Not Meet	Does Not Meet	3.3
Student H	EL	Does Not Meet	Does Not Meet	2.3
Student S	EL	Meets	Does Not Meet	3.5
Student T	EL	Does Not Meet	Does Not Meet	3.7
Student U	EL	Does Not Meet	Does Not Meet	3.3
Student D	Exited	Partially Meets	Does Not Meet	4.5
Student P	Exited	Partially Meets	Meets	4.6
Student R	Exited	Does Not Meet	Does Not Meet	4.6
Student I	Borderline	Does Not Meet	Does Not Meet	
Student L	Borderline	Does Not Meet	Does Not Meet	
Student M	Borderline	Does Not Meet	Does Not Meet	
Student O	Borderline	Partially Meets	Does Not Meet	
Student F	Hmong Speaker	Partially Meets	Meets	
Student J	Hmong Speaker	Partially Meets	Meets	
Student K	Hmong Speaker	Partially Meets	Partially Meets	
Student N	Hmong Speaker	Meets	Partially Meets	
Student Q	Hmong Speaker	Partially Meets	Partially Meets	
Student E	English Speaker	Meets	Partially Meets	
Student G	English Speaker	Meets	Meets	

As can be seen examining the initial data, there is a fair amount of variation of mathematical proficiency in each group, but some overall trends. 2 of the 7 ELs began proficient in math (28.6%), while the rest were scoring considerably below grade level. Meanwhile, 2 of the 3 Exited students (66.7%) scored close to proficient in math, while the third was below grade level. Only 1 of the 4 Borderline students (25%) was partially proficient in math, while the other 3 scored below grade level, which was much more in line with the official ELs. 1 of the 5 Native Hmong Speakers proficient in English (hereafter NHS) was fully proficient in math (20%), while the rest were partially proficient. Finally, both of the Native English Speakers (NES) scored on grade level in math. From this data, a considerable correlation with math testing scores and ELP can be inferred, considering that this reflects trends already established by other, more quantitative studies (National Assessment of Educational Progress, 2020). These trends will also manifest in interesting ways when considering the problem solving styles observed in the initial data gathering.

One student who clearly stands out from the other ELs was Student A. She was the only EL to score on grade level in both Reading and Math on both the NWEA/MAP test and the state standardized tests. It became known following the study that Student A's scores just barely qualify her to be exited from ESL services by state law. Her classification as an EL carried over from another state with a different threshold for considering students "proficient". All throughout the study, she will stand out from the other ELs and will more closely resemble the students in the "Exited" category, which she would have been classified under had school records reflected this at the time of the study.

Initial Data Gathering: Warm Up, Flipgrid Video and Test Question

As explained in Chapter Three, I began by administering a pre-assessment in the form of a warm up problem, which students were allowed to collaborate on, after which they explained their problem solving method in a short Flipgrid video, which was completed individually. I initially analyzed the students' written work on the warm ups paired with the videos to listen for evidence of whether students were following a problem-model approach, a direct translation approach, some hybrid of the two approaches, or something else entirely. I also looked closely at the first question on their individual tests to see how they approached a problem when not allowed to collaborate. A summary of this data is presented below:

Table 2

Initial Student Approaches to Problem Solving

Student	Category	Initial Approach
A	EL	Problem Model
B	EL	Mixed
C	EL	Mixed
H	EL	Direct Translation
S	EL	Mixed
T	EL	Mixed
U	EL	Mixed
D	Exited	Mixed
P	Exited	Problem Model
R	Exited	Problem Model
I	Borderline	Copying
L	Borderline	Copying
M	Borderline	Try Everything
O	Borderline	Intermediate

F	NHS	Intermediate
J	NHS	Problem Model
K	NHS	Intermediate
N	NHS	Problem Model
Q	NHS	Try Everything
E	NES	Problem Model
G	NES	Problem Model

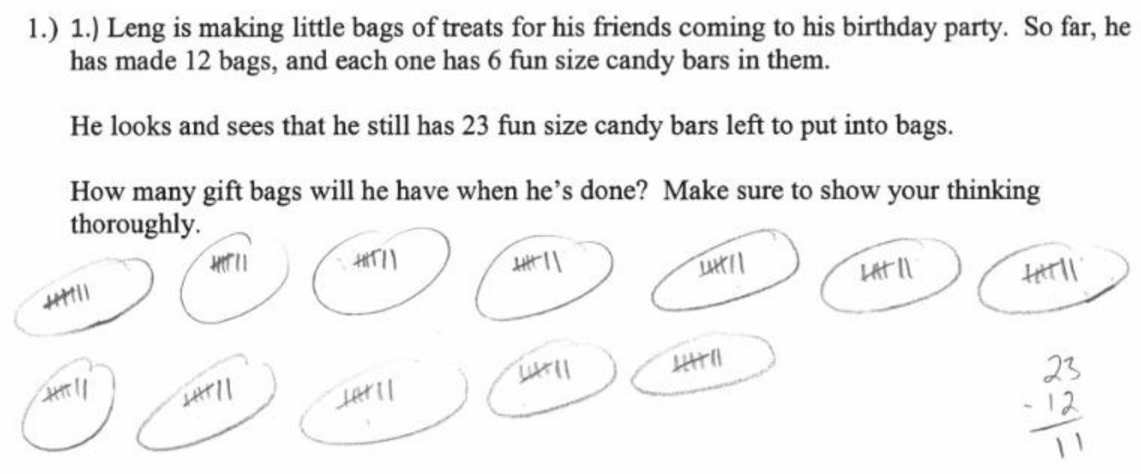
After revisiting the data at the end of the study, the warmups and accompanying videos showed 6 approaches, explained here. The first was the classic direct translation approach, in which the student would simply take the numbers in the problem and look for hints for what operation(s) to do with them, regardless of the story. Surprisingly, only one student (Student H) took this approach consistently on both the warm up and the test.

There were also the students taking the problem-model approach, in which every step taken is directly related to something happening in the story of the problem. These students both labeled their work clearly to imbue the numbers on the paper with meaning and/or described what they meant out loud in their explanations. There were a surprisingly high number of students already taking this approach, especially in the Exited (66.7%) and NES (100%) groups, as well as a significant number of the NHS group (40%). Notably, these groups also had higher rates of ELP. The only EL firmly showing a problem-model approach from the beginning was Student A, who was mentioned above.

One potential reason for the surprisingly high number of students already showing a problem-model approach and surprisingly low number of students showing the pure direct translation approach could be the fact that these students had already received a year of Problem Based Learning (PBL) in math class in sixth grade. This question needs more investigation to answer completely, though. It would be illuminating to conduct a similar study with a group of students in the first year of PBL implementation or a group of 6th graders, also doing PBL for the first time.

Also represented were students who did not completely fall into either of those two approaches, which manifested in two ways. One approach is described above as “mixed”, that is, the students would use the problem-model approach inconsistently. Sometimes these students would do some degree of modeling (either using clear meaning-making for some steps but not all, or all the way through), while other times they would use a direct translation, or even random approach, showing little to no evidence of meaning-making. This group was particularly prevalent in the EL group (71.4%), a little in the Exited group (33.3%) and not at all in the other groups. This could possibly be explained by these students employing a more problem-model approach when they understood the problem, and a direct translation approach when they understood it less.

The other of these two approaches is described as “intermediate”. These students would consistently begin the process in a problem model approach before ceasing to make sense of their steps, sometimes abruptly. A good example of this can be seen in the Student K’s response to the first test question, shown below:



1.) 1.) Leng is making little bags of treats for his friends coming to his birthday party. So far, he has made 12 bags, and each one has 6 fun size candy bars in them.

He looks and sees that he still has 23 fun size candy bars left to put into bags.

How many gift bags will he have when he's done? Make sure to show your thinking thoroughly.

Figure 9. Student K's response to a question on the first test.

The drawing she made clearly models the first piece of information: that is, that Leng has made 12 bags, each with 6 pieces of candy in it. She then nonsensically subtracts 23 and 12, potentially falling for the word "left", intended as an inconsistent language trap, which she seems to have fallen for at this point. The drawing is a clear example of modeling, while the subtraction is a clear example of number grabbing. The students following this approach were frequently found among the Borderline students (25%) and the NHS group (40%).

Another group that I anticipated coming across were the students whose main approach was simply copying off their neighbors. The two students doing this consistently (Student I and Student L) make up 50% of the students in the Borderline category. It was also harder to catch than expected. They did a decent job of disguising their copying, but the fact that they clearly did not understand the marks on their paper emerged by listening to their Flipgrid videos.

Finally, there was one approach I did not expect to find, which can best be described as trying every operation or method one can think of, looking at the results of all the different methods, and then choosing the one that seems to make sense. It is akin to the process of brainstorming. Sometimes the two students following this approach (one from the Borderline group, one from the NHS group) would select an answer that made sense, sometimes not. Student Q also made a habit of scribbling out all the discarded scratchwork that she decided did not make sense after trying them.

Initial Data Gathering: Achievement

I also looked at the first test in its entirety to get a baseline for the 21 participants' achievement in class. As explained in Chapter 3, the scores are on a 4 point scale, with 0 representing a completely blank test, a 1 representing minimal understanding, 2 representing a partial but incomplete understanding, a 3 representing a complete understanding but with slight errors, and a 4 representing a complete, well checked, well explained understanding. In the gradebook, a 0 is equivalent to scoring 40% on a traditional grading scale, and every point above that adds 15%. Therefore a 1 still translates to an F, a 2 is equivalent to a C-, a 3 is equivalent to a B, and a 4 being equivalent to the highest A. Their scores are given below, again, grouped according to their EL classification:

Table 3

Initial Classroom Test Scores

Student	Group	Test Score	Student	Group	Test Score	Student	Group	Test Score
A	EL	2.1	D	Exited	2.8	F	NHS	2.5
B	EL	2.2	P	Exited	2.9	J	NHS	2.7
C	EL	2.0	R	Exited	1.8	K	NHS	1.7
H	EL	1.6	I	Border line	2.1	N	NHS	3.5
S	EL	3.1	L	Border line	2.1	Q	NHS	1.7
T	EL	1.8	M	Border line	2.1	E	NES	3.4
U	EL	2.9	O	Border line	2.3	G	NES	3.2

It should be noted that this first test, as also noted in Chapter 3, was more procedural in nature than the following tests. This means that some students may have scored higher on this test than they would have on a more problem-solving heavy test. That said, the comparisons between this test and the following tests still went on to provide useful data for how different students' achievement in class did or did not change over the course of the study.

Initial Data Gathering: Attitude

Lastly, I gathered the students self-reported attitudes in response to the prompts "I have a strategy to help me make sense of word problems" and "I can explain my thinking in math class to others." The students responded by circling one of four emojis. I have recorded their responses by translating the most negative emoji to a score of 1 and the most positive emoji to a score of 4, and the others to 2 and 3. If a student circled both of the middle emojis, I entered the score as 2.5.

Table 4

Initial Attitudes Toward Problem Solving and Explaining Thinking

Student	Category	Solving	Explaining
A	EL	3	3
B	EL	1	1
C	EL	3	2
H	EL	3	3
S	EL	2	2
T	EL	2	1
U	EL	1	1
D	Exited	3	2
P	Exited	3	2
R	Exited	2	2
I	Borderline	2	2
L	Borderline	2	3
M	Borderline	2	2
O	Borderline	3	3
F	NHS	3	2
J	NHS	3	2.5
K	NHS	3	3
N	NHS	3	2
Q	NHS	2	1
E	NES	4	2
G	NES	3	3

Some definite trends can be noted immediately. First of all, it is noteworthy that only one student expressed more confidence about their ability to explain their thinking than their ability to make sense of a word problem. Secondly, the students in the EL category have a wide variation in these scores compared to the other categories. Also, in general, the borderline students mostly have consistently low confidence in their ability to solve problems, with the exception of Student O.

I also informally logged the responses of all my students to these questions and got average scores for the students as a whole. This data should be regarded with caution, since there is more variation in whose responses were turned in on any given day, but it can give a general impression of the classes' attitudes as a whole. The mean response to the question about solving problems was 2.51, indicating neutrality, while the mean response for the question on explaining was 2.14, indicating slightly negative feelings. The responses of the participants generally reflects the attitudes of their classmates.

Journal: Teaching the Intervention

My experiences teaching Read-And-Think (RAT) Math were recorded in my field journal the following week. The main thing I noted about teaching the RAT lesson was that the results were somewhat surprising. The classes that I expected to struggle more actually had relatively little difficulty drawing a diagram to represent the situation once I forced them to read it sentence-by-sentence. On the other hand, the class that I expected to struggle less got completely stuck. The problem that gave students clearly the most trouble was one that represented a division situation, where the character in the problem was attempting to saw a larger board into several smaller boards. The length of the long board and the length of each short board were both given, and students were tasked with figuring out the number of boards. It proved to be very difficult for the students to visualize the long board getting cut up, and that the short board would appear multiple times in the drawing instead of just once.

This experience suggested to me that my students probably struggled with multiplicative/divisive operational sense. It also made it clear that a single day of intervention might not be enough to make a lasting impact. Therefore I continued giving students problem solving warm ups similar to the ones I was using for my data. This also afforded me more opportunities to have my students practice using RAT on word problems that might continue to build their understanding of two step problems involving multiplication and division, as well as ratios. This may well have proved helpful, as the word problems built into the lessons I was teaching from the curriculum were mostly focused on building number sense with integers, rather than exposing students to a variety of situations in which to solve problems and interpret written information.

The first of these informal warm ups had the following prompt:

Solve the following problem using Read-And-Think. ***Make sure to draw a diagram.*** *Maria is buying beans from the corner store. Beans cost $1.49 for 1 can. She has brought $10 with her and her mom told her to get as many cans of beans as possible. She can use the rest to buy a snack. The snack she really wants is a snickers, which costs $0.51 for a fun size. What will Maria come home with?*

Here I got to see what the students did when explicitly told to draw a diagram, and when they encountered a word problem involving decimals. While a handful of participants in the study successfully solved the problem and included a diagram, the majority struggled to finish the problem. There were also a few participants who managed to solve the problem correctly but never drew a diagram. I then returned the graded warm ups to the students and provided more help with interpreting the language of the problem to draw the diagram, and also had students assist each other. This seemed to help the students further develop their understanding.

From this point on, I was mostly teaching the CPM curriculum without much modification, directing students to use RAT when they were stuck, as originally planned, until I reached the midpoint data gathering.

I am omitting most of the mid-point data I gathered as it mostly reflects the same trends that I saw on the final data gathering. Exceptions will be noted. Overall, I saw encouraging evidence that the students were making more sense of the word problems than before. I also took note of the whole class's responses to the survey questions. The question about having a method to make sense of word problems now had a mean response of 2.51 and the question about sharing one's thinking now had a mean response of 2.27. Interestingly, the score for feeling like the students had a strategy to make sense of word problems was *exactly the same* as before, while the overall confidence in explaining their thinking had gone up. From my observations, it seemed like many students were reacting to the increasing challenge of the math itself, and so some were beginning to get discouraged.

I should also note that around this time, the curriculum had moved on to cover the algebra of proportional relationships, which helped students develop tools such as tables and unit rates. Students also began seeing far more situations embedded in the word problems they were solving that were good for practicing RAT on. I also made a point of having students do warm ups about proportional relationships and unit rates following these lessons to give them more practice. This will be very relevant later, as it likely is tied to many students notably increasing the sophistication of the math they were using to solve these problems.

After students returned from their Thanksgiving break, I taught a second lesson on RAT to refresh the strategy and give more time to explicitly practice using it. Some of the problems in this lesson included fractions, which still proved to be a struggle for the students, but they eventually found success with them. I did notice, however, that the same class that struggled immensely with drawing a diagram of a division situation still struggled to recognize division in a word problem and got stuck on that problem.

Due to a lucky coincidence, shortly after the second RAT intervention day was the unit on problem solving already built into the CPM curriculum. This gave students still more practice drawing diagrams of situations, identifying the quantities to solve, and identifying the relationships between quantities. We also devoted a fair amount of class time to developing the tools to understand the subtleties in the language of the problems. Two phrases from one of these lessons in particular gave students a great deal of trouble. The first was a word problem in which students had to explain the difference between the following two problems:

- Myra has 15 marbles. This is 10 less than Dahlia.
- Myra has 15 marbles. Dahlia has 10 less marbles than Myra.

(CPM Educational Program, 2013).

Even though the textbook makes it clear that these are supposed to be different situations, almost *all* of the students drew both diagrams the same. The second problem involved a relationship in which one board was 7 inches longer than another length. Almost every single student drew a board and labeled the whole thing 7 inches. I also observed many students, including students who scored on grade level in math earlier and had been employing the problem model approach so far, struggling to recognize part-part-whole relationships in these problems and understand that when they had the whole and one part, they could use subtraction to get the second part. I believe that going through these experiences was instrumental to them continuing to develop the mathematics they needed to solve word problems.

Final Data Gathering: Problem Solving Approaches

At the end of study, the same assessment tools were given to the students for the third and final time. It should be noted that the problems themselves were different, but the mathematics they required remained very similar. On the next page are how the different students' approaches changed (or did not change) from the beginning of the study.

Table 5

Changes in Problem Solving Approach

Student	Category	Initial Approach	Changes to Approach
A	EL	Problem Model	Remained solidly problem model
B	EL	Mixed	Sometimes used strongly problem model, other times copied off neighbors
C	EL	Mixed	Remained Mixed, Increased use of labels and diagrams
H	EL	Direct Translation	Remained Direct Translation
S	EL	Mixed	Moderate move towards problem-model
T	EL	Mixed	Remained mixed in approach
U	EL	Mixed	Major move towards problem-model thinking
D	Exited	Mixed	Increase in problem-model approach, but not complete
P	Exited	Problem Model	Remained solidly problem model
R	Exited	Problem Model	Started and Ended Problem-Model after using Direct Translation at mid-point
I	Borderline	Copying	Move to Direct Translation
L	Borderline	Copying	Move to Direct Translation
M	Borderline	Try Everything	Slight change towards problem model
O	Borderline	Intermediate	Remained solidly problem model
F	NHS	Intermediate	Remained problem model but confused
J	NHS	Problem Model	Remained solidly problem model
K	NHS	Intermediate	Strongly increased problem-model before regressing slightly
N	NHS	Problem Model	Remained solidly problem model
Q	NHS	Try Everything	Significant move to problem model
E	NES	Problem Model	Remained solidly problem model
G	NES	Problem Model	Remained solidly problem model

When these results are analyzed according to their ELP grouping, the following trends appeared. The EL group saw 4 of the 7 participants (57.1%) either move towards or keep a problem model approach over the 10 weeks of the study (I am counting Student B in this, even though occasionally he still resorted to copying others), while 1 (14.3%) remained mixed in her approach, and 2 (28.6%) were still using ineffective approaches at the end. To elaborate on this, Student T's "mixed" style would alternate between very well-reasoned problem-model thinking on some problems, but resorting to number-grabbing on problems she understood less well. On the other hand, Student C's "mixed style" would alternate between an intermediate style and pure number grabbing, which is why I am counting it as an ineffective approach. However, it should be noted that she, at least, recognized the usefulness of labeling her numbers and drawing diagrams, even if her operations never began to make sense.

Student H stands out from all the other students in this study. He was the only student to only ever use a direct translation approach, and he also never moved away from this. He also never used operations that made sense in the context of the problems, and barely seemed to understand anything about the problem. For example, on the final test, the first question read:

> *"Eh Ler is making a tower out of marshmallows for a project in STEM class. After a few minutes, he checks on his progress. He sees that 48 marshmallows got him 8 inches for his tower. His teacher gave him 150 marshmallows to start with. Assuming his tower never falls, how tall will his final tower be at this rate?"*

Student H responded to this by multiplying 150 with 48 (incorrectly) and writing "it's that Eh Ler ate some marshmallow on the project in stem class. True." Following this study, I brought him up as a student who potentially should be evaluated for a learning disability. The results of this process were unknown at the writing of this thesis, so caution should be used when using his results to draw conclusions.

Moving on to the other groups, all 3 Exited students either maintained or moved towards a problem model approach, even if it was incomplete for Student D. Among the 4 Borderline students, 2 either maintained or moved towards a problem model approach (50%) while the other 2 actually started showing signs of a direct translation approach. However, the two students moving towards direct translation had also been copying others for most of the rest of the study, even on the test questions; in other words, this represented the first time I had observed them truly trying to solve problems for themselves instead of hiding their confusion. Finally, 100% of the students in the NHS and NES groups either maintained or moved towards a problem model approach by the end.

Final Data Gathering: Test Scores

I also compared their scores on the last test to the scores on the first and midpoint tests to see how students' achievement in math class changed over the course of the study. Their changes in achievement are listed in the table below.

Table 6

Changes Between Initial and Final Test Scores

Student	Group	Change	Student	Group	Change	Student	Group	Change
A	EL	1.7	D	Exited	0.2	F	NHS	-0.2
B	EL	0.1	P	Exited	0.1	J	NHS	0.9
C	EL	-0.6	R	Exited	0.7	K	NHS	0.2
H	EL	-0.6	I	Border line	-1.2	N	NHS	0.1
S	EL	0.2	L	Border line	-0.3	Q	NHS	1.2
T	EL	0.2	M	Border line	-0.2	E	NES	-0.5
U	EL	-1.6	O	Border line	0.7	G	NES	0.4

When examining these changes in test scores, I considered a change of 0.3 or less in either direction to be “about the same”, and anything greater than that in magnitude to be a significant rise or drop. The reason is that the scores are arrived at by totaling up the points on each of the ten questions (and each one is scored on a scale from 0 to 4), so a change of 0.4 would represent the portion of the final grade corresponding to a single question on the test. Anything less than that is essentially trivial. When examining these scores, it should also be noted that Student U spent most of the testing day in the nurse’s office, leaving most of his test blank, explaining his significant drop. If that drop is ignored, he shows a drop from 2.9 to 2.5 (still significant, but not nearly as drastic, and still possibly explicable by distraction due to his illness).

The EL group does not show a great deal of change in achievement. One student showed a significant increase (14.3%), 3 remained about the same (42.9%), and 3 had significant decreases (42.9%). It is interesting that 2 of the 3 students who had significant decreases *also* did not adjust their approaches towards a problem-model approach, while the third is Student U, whose situation is noted above. The Exited group had 2 students remain about the same (66.7%) and one student significantly increase in achievement (33.3%). The Borderline group had one student significantly increase her score (25%), 2 remain about the same (50%) and one significantly drop (25%). Again, the student who significantly increased was a problem-model user, while the one who significantly dropped was a copier who moved to a direct translation approach. The NHS group had 2 students significantly increase (40%) and the rest remain about the same (60%). Lastly, the NES group had one student (50%) significantly increase and the other (50%) significantly decrease.

The groups whose achievement seems to have been impacted the strongest were the Exited and NHS groups. Their mean increases in achievement were 0.33 and 0.44, respectively, and the students in those groups with the greatest increases either developed or maintained a problem-model approach over the course of the study.

Final Data Gathering: Survey Data

The final piece of data gathered and compared at the end was the survey data on how the students felt about having a strategy to make sense of word problems and explaining their thinking out loud in math class. The data below represents how much and in what direction their answers changed from the beginning of the study to the end. An entry of +3 would indicate going from the most negative response to the most positive response.

Table 7

Changes to Attitudes Towards Problem Solving and Explaining Thinking

Student	Category	Changes in Solving	Changes in Explaining
A	EL	0	0
B	EL	+3	+3
C	EL	-0.5	+0.5
H	EL	0	0
S	EL	+1	+1
T	EL	0	0
U	EL	+3	+2
D	Exited	0	+1
P	Exited	0	+1
R	Exited	0	0
I	Borderline	+1	0
L	Borderline	+0.5	0
M	Borderline	+1	0
O	Borderline	0	-1
F	NHS	-1	+1
J	NHS	+1	+0.5
K	NHS	0	0
N	NHS	0	+1
Q	NHS	+1	0
E	NES	-1	0
G	NES	-1	-0.5

Examining the EL group, 3 of them (42.9%) showed significant increases in positivity about both solving and explaining, while the other 4 (57.1%) showed little to no change. The mean changes were +0.93 for solving, and +0.93 for explaining. The Exited group showed no change around solving, but 2 of the 3 (66.7%) showed an increase in positivity around explaining, while the third (33.3%) remained the same. The mean changes for this group were 0 for solving and +0.67 for explaining. In the Borderline group, 2 of the 4 showed a significant increase in positivity around solving (50%), while the other 2 (50%) showed little or no change. In the same group, no students increased their positivity around explaining, and one (25%) even decreased in positivity. The mean changes in the Borderline group were +0.63 for solving and -0.25 for explaining.

In the NHS group, 2 students (40%) increased their positivity around solving, while 1 (20%) decreased and 2 (40%) showed little or no change. In the same group, 2 (40%) increased their positivity around explaining, while the other 3 (60%) either showed little or no change. The mean changes in this group were +0.2 for solving and +0.5 for explaining. It should also be noted that this group already started off with 80% of the students already professing positive attitudes about solving, unlike the Exited group, which was uniformly negative. Lastly, in the NES group, both students (100%) decreased in positivity around solving, and showed little or no change around explaining. The mean changes for this group were -1 for solving and -0.25 for explaining. The decreases in this group might be explained by a phenomenon Jo Boaler (2016) writes about, in which students who normally achieve highly in math suffer setbacks in confidence when confronted with math that is difficult for them, which would be consistent with their responses to the warmup they did shortly before responding to this survey.

Again, I also informally tallied up responses from the whole class (or, rather, the 65 students who remembered to fill out the survey) in the field journal. The solving question now had a mean score of 2.61, an increase of 0.1 from both mid-study and the beginning - equivalent to 6.5 students raising their response by 1 point. The explaining question now had a score of 2.37, showing an increase of 0.1 from mid-study and 0.23 from the beginning of the study, or equivalent to about 15 students raising their rating by 1 point.

Interestingly, the participants of the study had higher mean increases in ratings from the beginning of the study: 0.38 in solving and 0.45 in explaining. Part of the discrepancy is explained by the inclusion of my 3rd period class in these totals, from which none of my participants came. This class showed notably less change in attitudes than the other two. This raises questions about what made this class different from the others, but the fact that so few students from that particular class volunteered to participate in the study might be related, either potentially explained by motivation or confidence or both. Further research would be required to discover answers to these questions.

Examining the Data When Sorted by Initial Problem Solving Style

The data above is somewhat illuminating. However, a much clearer picture emerges when it is re-sorted according to the problem solving approaches the students took at the start of the year. This approach to interpreting the data groups students who underwent similar struggles and similar changes together. We will examine these groupings, beginning with the most solidly ineffective approaches and proceeding to the more effective ones.

Exactly two students began with the approach of copying their neighbors, and both of them had a fair amount in common. Both girls spoke Hmong at home but scored well below their peers in both reading and math, and so they were classified as "Borderline". They also both persisted in copying until the end of the study, at which point they began to actually struggle through problems, even if that meant most of their solutions took a direct translation approach. Interestingly, both of these students increased their positivity about having a strategy to make sense of word problems by the end, but did not change their attitudes about explaining their thinking (one was a 2, so she remained negative, the other a 3, remaining positive). One of their test scores dropped *very* significantly (-1.2) while the other remained about the same, for a mean drop of -0.75. They also never increased the sophistication of the mathematics they used to solve the problems. This seems to suggest that the intervention was not very effective for these students within the time frame of the study.

Secondly, there was only one student who began the study with a pure direct translation strategy. This student (Student H) has already been discussed above as to why his data should be examined with caution due to being sick while taking the last test. That said, it is clear that his approach barely changed, aside from introducing more narrative elements into his explanations, and neither did his attitudes (which remained 3s for the duration of the study), and nor did the sophistication of his mathematics, which never made any sense to begin with. His achievement dropped significantly. As such, it is safe to say that the intervention was also not successful for him within the duration of the study.

The third group was composed of two students whose approach could be described as trying everything they could with the numbers in the problem, then selecting the results that seemed the most reasonable. Both of these students spoke Hmong at home, but one was classified as Borderline (Student M), while the other student's reading scores were close to on grade level (Student Q) and was classified as NHS. Both shifted their approach towards a problem model approach by the end of the study; M's was partial, still retaining some elements of his original approach, while Q's transition was complete. Both also increased their positivity about feeling like they had a strategy to solve word problems by 1 point, but both of them still felt negative about explaining their thinking by the end. One of them *very* significantly increased their test scores (+1.2) for Student Q, while Student M remained about the same (-0.2), for a mean increase of +0.5. Student Q also significantly increased the sophistication of the mathematics she used, especially around multiplicative reasoning, and both significantly improved how they organized the information from the problem. The intervention appears to have been successful for students in this group in both their approach, their attitudes, and their achievement.

There were 6 students who began the study with a mixed approach (switching between a direct translation approach and either an intermediate or problem-model approach). Notably, 5 of them were ELs, and the last was an exited EL. Also, this group is unique in terms of being made up mostly of boys (4 out of 6, or 66.7%). Among these 6 students, 1 (Student C) continued to use operations that made no sense, but did start using diagrams and labels, suggesting she was at least *trying* to change her approach, but was impeded by a lack of operational sense. Another (Student T) remained mixed in her approach, in that she was still sometimes strongly using direct translation, while other times strongly using problem-model. The other 4 made either moderate or major progress towards using a problem-model approach.

These students had a mixture of how they felt about having a strategy for problem solving. However, C and T, who did not change their approach, also did not increase their confidence about having a strategy (T stayed the same, and C changed by -0.5). Of the four that did change, 3 of them significantly increased their positivity about having a strategy, and 2 of them drastically so. The mean change about solving was +1.08 for these 6 students. Explaining their thinking, though, showed even greater change. of them increased their response about explaining their thinking, with a mean change of +1.25. Curiously, the one who showed no increase in positivity here (Student T) actually showed some of the *strongest* growth in explaining her thinking, but it was not accompanied by an increase in positive feelings around it.

Students who began with a mixed approach did not show much change in their achievement, however. 2 significantly dropped, although one of these was Student U, discussed above, and the other was Student C, whose approach did not change. The other 4 students did not change significantly, for a mean change of -0.25, or -0.05 if Student U's score is adjusted. That said, of the 4 students who moved towards a problem-model approach, 3 increased the sophistication of their mathematical techniques on the word problems, particularly with multiplicative reasoning, unit rates, and even tables. The fourth already was using these tools from the start. With all this in mind, it appears that the intervention was *mostly* successful with students beginning with a mixed approach in all areas except their academic achievement.

There were also 3 students who began with an intermediate approach in that they always attempted to model the problem at first, but would often abandon the model partway through the process for a more procedural/less reasoned approach. All 3 were girls who spoke Hmong at home; none were identified as ELs, but one (Student O) was put in the Borderline group because of her reading scores. All 3 moved towards the problem model approach by the end, although Student F would still sometimes have her sense-making break down when confused. All 3 began rating having a strategy for solving problems as 3, and 2 also rated explaining as 3, while Student F responded with a 2. Student F decreased her rating for solving to a 2 by then, but increased her rating for explaining to a 3. Student K's ratings remained the same, and Student O's rating for solving also remained a 3, but her rating for explaining dropped to a 2, despite very noticeably *improving* at this skill. 2 of these students' achievement remained about the same while 1 significantly increased for a mean change of +0.23. 2 of these students also significantly increased the sophistication of the mathematics they used to solve the problems, while Student F's mathematics remained about the same. All things considered, the intervention appears to have been moderately successful for this group of students, particularly in terms of problem solving approach and mathematical sophistication.

Lastly, there are the 7 students who began the study already using a problem-model approach on all problems. This group included one EL (Student A), two exited ELs (Students P and R), two native Hmong Speakers who were never ELs (Students J and N) and both of the native English speakers (Students E and G). This group was dominated by girls, but there were 2 boys (R and G). Unsurprisingly, all of them retained their problem-model approach by the end, although Student R did momentarily move into a mixed approach at mid-study, but he had returned to problem-model by the end. 6 of these students (85.7%) already professed positive feelings about having a strategy to make sense of problems, but 5 of them (71.4%) felt negatively about explaining their thinking out loud. As noted before, both of the NES students decreased their positivity about having a problem solving strategy, while 1 (14.3%) increased her positivity, and the rest (57.1%) remained the same, resulting in a mean change of -0.14 on this question. 4 of them (57.1%) remained the same or slightly decreased their positivity about explaining their thinking, and the rest (42.9%) increased their positivity about explaining, resulting in a mean change of +0.29 on this question.

While this group did not show much significant change in approach or attitude (despite overall feeling slightly more positive about explaining their thinking), they did show much more change in their achievement and mathematical sophistication. While 1 experienced a significant drop in test scores (-0.5) and 2 remained about the same, the other 4 all showed significant increases ranging from +0.4 to +1.7. The mean change on test scores for this group was +0.49, representing a significant overall increase. Only one of these students (Student E) did not noticeably increase her mathematical sophistication (which, to be fair, was already higher than that of most of her peers, beginning the study already using unit rates and multiplicative reasoning). The others all adopted using unit rates by the end, and one (Student J) showed a drastic increase in sophistication, having started the study often showing rather disorganized work on her paper, but ending with well-organized work exhibiting efficient multiplicative reasoning employing unit rates. With all this in mind, it appears the intervention, which was not even intended to help students like this, actually was associated with development of more sophisticated mathematical tools and higher achievement as shown on test scores.

Examining Explanation Style and Confidence in Explaining

One curious trend that emerged from the data was that multiple students who noticeably improved at explaining their thinking in the Flipgrid videos did not report feeling more positive in explaining their thinking, or even lowered their ratings. That caused me to also organize the data according to how the students explained their thinking at the start of the study and how their explanation style changed and compared it to how their feelings about explaining their thinking changed.

7 students began the study giving highly procedural explanations in the videos. Noticeably, this group was almost entirely ELs with 2 Borderline students. 1 of these students (Student L) remained procedural in her explanations, and her feelings remained positive. 2 of them (C and H) never made sense in their mathematics, but their explanations did begin to incorporate more narrative elements, and this was accompanied by a rise in positivity in both. The other 4 (B, S, T and O) all completely changed their explanation styles to show a high degree of modeling (i.e., labeling numbers, connecting steps to things happening in the story, referencing the situations, etc.). Of these 4 students, the two boys (B and U) both went from feeling extremely negative to feeling extremely positive about explaining, while one girl (T) remained feeling extremely negative about explaining and the other girl (O) went from a 3 to a 2.

4 students gave procedural explanations, but would always put their answers back in context after listing the steps. This group included 1 exited EL, 1 native English speaker and 2 students from the Borderline group. 1 of them (Student I) actually became fully procedural in her explanations, which also matched her shift in problem solving style, and still felt negative by the end. Another (Student M) kept the same explanation

style, and also stayed negative in his feelings about explaining. The other 2 began including more modeling in their explanations of the steps; one of them went from feeling negatively about explaining to feeling positively, while the other one decreased his rating from a 3 to a 2.5. This last student explains in his video that he rated it like this because the problem itself was harder for him.

Lastly, 10 students always used a high degree of modeling in their explanations, referencing the story of the problem when they talked about the steps they took. 6 of them stayed about the same in their explanation style by the end, but 4 of those 6 ended up showing positive feelings about explaining their thinking by the end. The other 2 remained feeling negative (using a rating of 2). Finally, the other 4 students starting with this style actually became even highly modeled in their explanations by the end. Of those 4 who improved even more, 3 of them increased their feelings about explaining from negative to positive, while the last (Student Q) kept her rating at a 1 the whole study.

To summarize, 10 students began using significantly more modeling when explaining their thinking. All but one started off with negative feelings about explaining their thinking. By the end of the study, 4 of them either *remained negative or became more negative* in their feelings, while the other 6 felt more positive by the end. 14 of the 21 participants began the study reporting negative feelings about explaining their thinking; 12 of them (85.7%) became significantly more modeled in their explanation style by the end of the study, but only 8 of them (57.1%) showed an increase in positive feelings about explaining their thinking. 3 of the 4 students who became more modeled in their explanation style but stayed or became more negative in their feelings were girls. On the other hand the 2 students who went from the most negative feelings about explaining to the most positive were both boys. This raises interesting questions about how students of different genders feel about explaining their thinking in math, regardless of how much they improve at it, as well as how to support girls with developing more confidence in speaking about mathematics. This study was not intended to address these questions directly, so further research is required.

Conclusion

This chapter has presented the data collected over the course of this study. To summarize, the following changes appeared in the 10 weeks of implementing the RAT intervention. The 3 students who either copied off their neighbors or initially used a pure direct translation approach showed little to no change in problem solving approach, got lower test scores overall and did not adopt more sophisticated mathematical tools when solving problems. Of the 11 students who initially used the “try everything” strategy, mixed strategies or used an intermediate approach, 81.8% had at least partially changed their approach towards the problem-model approach, 63.6% felt positively about having a strategy to help them make sense of word problems, and 63.6% had significantly increased the sophistication of the mathematics they used to solve word problems. However, only 27.2% significantly increased their achievement on the math tests. Of the 7 students who had always used a problem-model approach, 100% of them maintained this approach by the end of the study, 57.1% significantly improved their achievement on math tests, and 85.7% significantly increased the sophistication of the mathematics they used to solve word problems.

Finally, 76.2% of the students in the study finished the study using modeling in their explanation style by the end of the study, but whether this was accompanied by an increase in positive feelings about their ability to explain their thinking was inconsistent, and seemed to be related somewhat to the genders of the students. Chapter 5 will discuss major findings from the study, implications for teaching, the limitations of the study, professional growth and insights, and recommendations for further research.

CHAPTER FIVE

Conclusions

This chapter will present the major findings, discussion on these findings, implications for teaching, limitations of the study, my own professional growth and insights, further recommendations for research, and how I intend to communicate and use the results of the study.

The purpose of this study was to examine how students of varying levels of English Language Proficiency (ELP) responded to being taught and encouraged to use the Read-And-Think (RAT) math strategy for solving mathematical word problems. It was intended to answer the research question: *how do students with varying levels of English proficiency respond to identified teaching strategies noted in the research literature that support them with developing a "problem-model approach" to solving mathematics word problems?* Specifically, I wanted to see if English Learners (ELs) and other students who speak a language other than English at home would develop a problem-model approach to solving word problems, that is, an approach in which mathematical operations are imbued with meaning in relation to the story. This is opposed to approaches like the direct translation approach in which students perform operations on the numbers in the problem without regard to making sense of what they are doing.

Major Findings

The major findings of the study are nuanced, but mostly positive. Students who began the study sometimes resorting to the direct translation approach, while other times making more sense of the problem, mostly adapted their approaches to the problem-model approach, even if not completely. These students also generally felt significantly more positive about solving word problems and explaining their thinking out loud, and also increased the sophistication of the mathematical tools they used to solve word problems. Students who already used a problem-model approach (who generally also had higher levels of ELP and reading scores than the other students just mentioned) increased their math classroom test scores and the mathematical sophistication of the tools they used, and also generally felt more positively about explaining their thinking.

The four students who did not seem to benefit much from the intervention were those whose work suggested a severe lack of understanding behind the meanings of different operations from the beginning. Two of those students masked their lack of understanding by constantly copying off their neighbors, but who *did* start productively struggling by the end of the study. The other two seemed to learn that their work and explanations should contain narrative elements, but the operations they chose always seemed more-or-less random. Three of these four students showed a significant drop on their classroom test scores.

Discussion

To answer the research question, it appears that most learners responded to being taught the RAT math strategy by moving towards or maintaining a problem-model approach to solving word problems, gaining confidence in explaining their thinking in math class, and increasing the sophistication of the mathematical tools they used for solving these problems, especially when it came to multiplicative reasoning (as opposed to using exclusively repeated addition or subtraction to solve problems). This is largely consistent with the assertions made by the developers of the RAT Math (Nessel & Graham, 2006)/Mathematical Bet Lines (Dick et al., 2016) strategies outlined in Chapter Two: that using this method prompts students to think about math problems as stories, and also prompts them to ponder how the information given by a problem relates to the information that can be deduced from it using mathematical operations. The other positive outcomes were not explicitly predicted by the literature, but are consistent with the asserted benefits.

The main requirement for making these moves seemed to be that students had a certain baseline understanding of the meanings of different operations. Those who never demonstrated this understanding from the beginning of the study (and whose MAP test scores indicated around a 3rd grade level of mathematical proficiency) either retained their initial approaches or moved *towards* a direct translation approach. It is unclear whether the underlying reason for this is their schematic understanding of math operations, their cognitive academic language proficiency, an undiagnosed learning disability, or a combination of the above. However, the extreme discrepancies in mathematical modeling between the 2 ELs in this category and other ELs in the class with similar WIDA scores suggests that their ELP is likely not solely to blame. For the two students who moved towards direct translation, it is possible that they were simply revealing the approach they already would have had if they had not been copying off of their neighbors. This suggests that more intensive interventions, such as Schema Based Instruction (SBI, discussed in Chapter Two), might be more helpful for students like this.

This is also consistent with other literature discussed in Chapter Two, particularly with regards to the targeted populations of the different strategies. Some authors, notably Orosco (2014) and Griffin and Jitendra (2008), specifically targeted their interventions to students with Mathematical Difficulty (MD), and they both asserted the need for more direct instruction when intervening with these students. This study never was intended to address MD directly, nor did it attempt to classify students as having or not having MD, and it certainly did not implement direct instruction approaches with such students. However, it is an open question as to whether the approaches described above perhaps should be used for the students that did not show the benefits described earlier, or whether these students simply needed more than 10 weeks to begin showing them.

It is also important to consider the possibility that part of why most of the participants of this study improved so much was that simply collecting the data required setting aside time for multi-step problem solving and explaining their thinking in video form. Every time this data was collected, it increased the amount of time we spent solving problems that did not rely on understanding rational numbers, instead focusing their attention on the four primary operations of addition, subtraction, multiplication and division and using them to model real world situations. It also forced students to explain their reasoning out loud in a way that was harder to opt out of than typical group discussions, by requiring them to take videos of themselves explaining their process.

Implications for Teaching

I believe that the implications of this study for teaching are as nuanced as the results. First and foremost, it demonstrates that math teachers should set aside time to explicitly state the expectation that students read word problems *as a story* and use that to inform the mathematical decisions they make. It also demonstrates that setting aside time for problem solving and explaining their thinking benefits both ELs and non-ELs alike, but in different ways.

On the other hand, it also demonstrates that teaching the whole class RAT math is not sufficient to help all students. There is a certain small but important part of the population of students whose operational sense is so under-developed that, even though they might show the *intention* of using a problem-model approach they still select operations nonsensically. This implies to me that more focused small-group interventions that specifically target operational sense might need to be used alongside whole class interventions like RAT, and administrators should provide either push-in or pull-out support to enable these time-intensive interventions to happen if they want to encourage their most struggling students to develop numeracy.

Limitations of the Study

Many of the limitations of this study were previously discussed in chapter three, including the relatively small convenience sample, the fact that the participants' teacher was also the researcher, introducing inherent bias into any conclusions drawn, and the relatively short time-scale of the study. I believe it is entirely possible that ten weeks might not be enough time to see how students develop in their approaches, like those who only just started to struggle productively at the end of the study.

Another limitation of the study is the general linguistic homogeneity of the participants: 18 of the 21 students spoke Hmong at home. Only 1 participant spoke another language (Karen), and she notably responded to the intervention differently from the other ELs. This limits the generalizability of the study even more than the other factors above.

Another limiting factor is the fact that students were given the option whether or not to have their data included in the study; while this was deemed ethically necessary, it probably did result in a selection bias. This became clear in comparing the changes in feelings about problem solving among the study's participants as opposed to the whole class's. While nothing conclusive can be said as to how or why this has affected the results, one can speculate that the students who opted into the study also showed a higher level of investment in improving their problem solving than those who did not opt in.

One last limitation seems to be that gender diversity and ELP were not very balanced among the participants. Interestingly, most of the boys in the study were either ELs or exited ELs. Most of the participants who spoke Hmong at home but were close to being on grade level in reading were girls. The interactions of problem solving style and gender emerged in the data, which leaves me with little idea of how boys who spoke another language at home but had strong reading skills would have responded to the intervention.

Professional Growth and Insights

Conducting this study has helped me to grow as an educator by requiring me to set aside the time to deeply examine my students' approaches to solving problems. I have learned much about the patterns of students' strengths and struggles when it comes to mathematics and reading (for instance, realizing that most of my students began seventh grade only feeling comfortable modeling situations with addition and subtraction rather than multiplication and division) and thus how to better support their development as mathematicians. I also have become more intentional and informed about how to teach and support literacy in a secondary math classroom.

Examining the data closely also gave me insights into how the gender divide affects my students' experiences in the classroom, which was never the original intention of the study, but which emerged nonetheless. It made it clear to me that many of my female students face confidence issues in mathematics classrooms. While this knowledge was perfectly consistent with what I had already read about the issue, such as in Jo Boaler's work (2016), realizing that improving one's ability to explain one's thinking often did not make my female students feel more positive about explaining their thinking in math class was startling nonetheless. This insight has helped me to consider how better to support these students in creating safe spaces for them to share their thinking.

Further Research Recommendations

While this study has been illuminating, many more questions remain to be researched. For one, it remains to be seen how students would respond to the RAT math strategy during their first year in a Problem Based Learning (PBL) classroom, such as sixth graders at my school. It would also be helpful to know how elementary school EL students might respond to the strategy and if it might help address the severe lack of operational sense seen in some students by the time they reach 7th grade. It also remains to be seen how a more diverse population of ELs would respond to the strategy. Finally, the question remains how best to help the small group of students who did not move towards a problem-model approach by the end of the study, and whether one of the other problem solving strategies discussed in chapter two would have helped them, such as SBI.

Communicating and Using Results

The administrators and instructional coaches at my school expressed an interest in hearing about the results of this study when completed, especially as they had been considering training and coaching teachers in using a strategy like RAT on a wider scale at the school. After all, RAT/Mathematical Bet Lines were adapted specifically from reading comprehension strategies developed for use in teaching ELs, as discussed in Chapter Two, and my school was designed to meet the needs of a high EL population. I intend to compile a concise report on my findings for the Director of Teaching and Learning at the school and set up a meeting to communicate them. I intend to strongly recommend that the school proceed with training the elementary and middle school teachers in teaching this strategy and offering my help in this process, if they take my recommendations. Even if the school does not make a concerted effort towards implementing this on a wide scale, I will still offer to teach workshops on implementing the strategy at the schools professional development days.

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Appendix A

Initial Test Questions

1.) Leng is making little bags of treats for his friends coming to his birthday party. So far, he has made 12 bags, and each one has 6 fun size candy bars in them.

He looks and sees that he still has 23 fun size candy bars left to put into bags.

How many gift bags will he have when he's done? Make sure to show your thinking thoroughly.

2.) $-15 + (-20)$ has the same result as which of the following?

A. $15 + 20$ B. $-(15 + 20)$ C. $15 + (-20)$

D. $-[(-15)+(-20)]$ E. None of these.

3.) **Calculate** the following, then place them in **order from least to greatest**.

a. $-12 + 6$

b. $-3 + (-7)$

c. $-3(-3) + 4(-3)$

4.) Show a diagram of +'s and –'s that each expression could represent then find the value of the expression.

a. $-7 + 6 + (-2) =$

b. $3(-2) =$

c. $4 + (-5) + 6$

d. $5(-3)$

5.) For each sequence, what are the next three numbers in the pattern? Explain the pattern in words.

a. 8, 12, 16, 20, … b. 5, 12, 19, 26, … c. 14, 11, 8, 5, …

6.) Simplify:

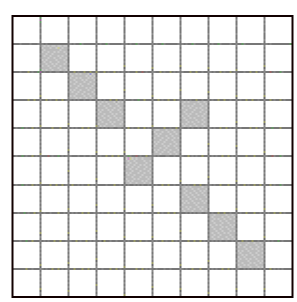
a. $(-4)(-5)(-2)$

b. $(-5) + (-5) + (-1)$

c. $68 + 7 \cdot (-6)$

d. $4(-6) + -6$

7.) Consider the representation below. Write the portion shaded as



a. a fraction.

b. a decimal.

c. a percent.

8.) Simplify.

a. $\frac{4}{5} \cdot (0.8) =$

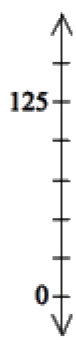
b. $\frac{3}{5} \cdot \frac{5}{7} =$

c. $\frac{4}{9} \cdot \frac{3}{10}$

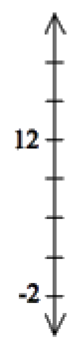
d. $\frac{7}{8} \cdot 2\frac{1}{8}$

9.) Complete the scales on the number lines.

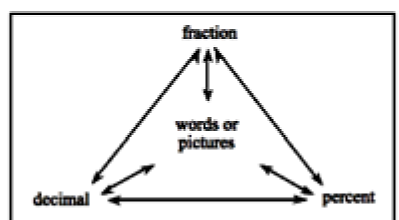
a.



b.



10.) At right is a Representations of a Portion web. Complete the web to fit the situation represented by this quote from an article:



Representations of a Portion

"*Youth exposure to alcohol advertising on U.S. television increased 71%...*"

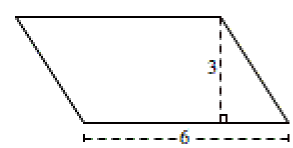
Appendix B

Midpoint Test Questions

1.) Solve the problem using any method, but be sure to *show your thinking!*

Pachia is helping her dad plant a garden. Together, they cleared out a rectangle of earth that is 4 feet by 11 feet. Her favorite food is tomatoes, so she googles how much space a tomato plant needs, which turns out to be 9 square feet. How many tomato plants will she be able to plant? Will there be space for anything else?

2.) Consider the parallelogram at right.



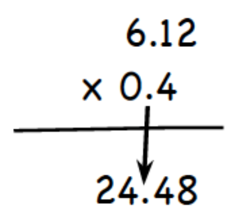
a. What is the length of the base of the parallelogram?

b. What is the length of the height of the parallelogram?

c. What is the area of the parallelogram?

d. Draw a rectangle that would have the same area as this parallelogram. Include the dimensions on the rectangle!

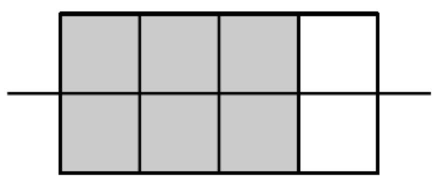
3.) At right is Jasmyn's work where she was multiplying two decimal numbers. Did she do the problem properly? If so, how do you know? If not, what did she do incorrectly? Be clear and complete.



4.) Julie just doesn't get how fraction division works. *"None of it makes senses!"* she whined to Scott one day. *"I mean, I know that because I can do all that 'invert and multiply' stuff. But WHY?!? I don't understand why that works!"*

Scott sighed heavily. Then he grabbed Julie's pencil and paper and drew this picture:

When he flashed the drawing back at Julie she said *"Yeah? So what? That's a picture of three fourths. I'm not stupid, you know!"* Scott pulled the paper back quickly and added one thing to the picture. He pushed the paper back at Julie, got up and left. Here is what the picture now looked like:



Julie is dumbfounded. *"What is that supposed to do for me?!"* She yelled after Scott as he walked away. Help Julie out. Either explain what Scott was trying to illustrate for her with the diagram, or use your own method to explain to Julie why it makes sense . Note: Do not say, *"Just invert and multiply!"*

5.) **Why** does -6 - (-2) equal -4? Be clear and complete.

6.) **Why** does -3·(-2) equal +6? Be clear and complete.

7.) What is $\frac{9}{8}$ written as a percent? How do you know? Explain.

8.) Compute each of the following.

a. $3\frac{2}{7} \cdot 5 =$

b. $\frac{5}{8}$ of 12 =

c. (8 + (-4))(-3 - 2) - 3(5)=

d. $6.2 \div \frac{3}{4} =$

e. 4(6 + -3) + 3(4)=

f. -2(6) + 3(5 - 4·2)=

9.) As a challenge, Zevel told his friend Rian that he wanted to see if he could rewrite $\frac{3}{7}$ as a decimal without using his calculator. The next day Rian asked *"So, how did it go, Zevel? Did you rewrite* $\frac{3}{7}$ *as a decimal?" "Nah,"* he replied. *"Didn't work,"* he added. *"What do you mean it didn't work?"* she asked. *"I tried dividing and the decimal never terminated and never repeated. I guess it just can't be written in any nice decimal way."*

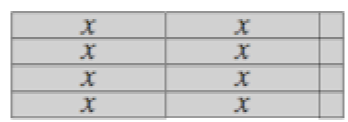
Help them out: do you think Zevel might have made a mistake? Explain your response. Is there way to tell if the decimal will terminate or repeat? Explain. Show Rian and Zevel (and your teacher) everything you know about decimals, repeating and terminating.

Appendix C

Final Test Questions

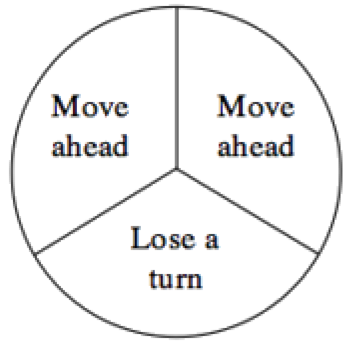
1.) Eh Ler is making a tower out of marshmallows for a project in STEM class. After a few minutes he checks on his progress. Sees that 48 marshmallows got him 8 inches for his tower. His teacher gave him 150 marshmallows to start with. Assuming his tower never falls, how tall will his final tower be at this rate?

2.) Write two algebraic expressions for the area of the algebra titles shown. One of your expressions should show the distributive property, while the second should not.



3.) Scott was considering the equation $5 - x = 12$. He said, "That seems impossible. How can I start with 5 and subtract from it and have more than 5 as the difference?" Write a note to Scott explaining what the solution for x is and why it is possible.

4.) Marcia just bought a game that came with the spinner below. On each turn, the player rolls a die and then spins the spinner. The spinner determines if the player actually gets to move.



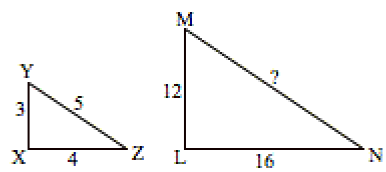
Note: the pieces on the spinner are all the same size.

a. What is the most likely outcome with this spinner? Why?

b. What is the probability of losing your turn? Explain.

5.) Triangles XYZ and LMN are similar.

a. What **scale factor** was used to enlarge triangle XYZ to create triangle LMN? Explain how you know.



b. What is the ratio of side lengths of triangle XYZ to triangle LMN? Write it as a fraction and simplify.

c. Calculate the length of the missing side of triangle LMN. Show your work.

6.) Complete the table below (Simplify fractions if possible):

Fraction	Decimal	Percent
	0.06	
		0.5 %

7) Simplify.

a. $\frac{3}{8} - \frac{1}{4}$

b. $2\frac{1}{3} \cdot 4\frac{4}{5}$

c. $0.2 - \frac{1}{10}$

d. $\frac{3}{8} \cdot \frac{1}{5}$

e. $10 \cdot \frac{1}{2}$

f. $6.57 \cdot 2.8$

8.) According to the scale on a map, 1 inch on the map = 16 miles. How many inches on the map would represent 24 miles?

9.) Tim is dividing up a $2\frac{1}{2}$ pound bag of nuts into smaller portions.

Hint: try drawing a diagram to help you solve the problem!

a. How many $\frac{1}{4}$ pound bags of nuts can be made from a $2\frac{1}{2}$ pound bag of nuts?

b. How many $\frac{1}{2}$ pound bags of nuts can be made from a $2\frac{1}{2}$ pound bag of nuts?

c. How many $\frac{3}{4}$ pound bags of nuts can be made from a $2\frac{1}{2}$ pound bag of nuts?

10.) A copy machine produces 40 copies in 5 minutes. How many copies can the machine make in 30 minutes and how do you know?

Appendix D

Warm Up Questions

Initial Warm Up

James is making pancakes. He knows that 120 ml of milk will make 8 pancakes, and he notices he has 400 ml of milk left in the jug. How many pancakes will he be able to make?

Solve this problem using any method you like, but make sure to show me HOW you solved it.

Midpoint Warm Up

Kou just bought a brand new fish tank and is ready to make a home for his new fish. He looks on the box and sees the tank can hold 13,000 ml. After he puts 3000 ml of gravel and plants on the bottom, he starts filling it with water. His water jug can hold 750 ml of water. How many times will he have to fill up his jug to fill up the tank?

Solve this problem using any method you like, but make sure to show me HOW you solved it.

Final Warm Up

Sue is saving up for a new Xbox. He checked at the store and saw that it will cost him $249. He already has $31 saved up. He will earn the rest by babysitting for the neighbors. He remembers that last time they paid him $45 for babysitting 3 hours. How long will he need to babysit to get enough money to buy the Xbox?

Solve this problem using any method you like, but make sure to show me HOW you solved it.

Appendix E

Consent Form in English

Informed Consent to Participate in Research

Your child is being asked to participate in a research study. This form provides you with information about the study. Your child's teacher will answer all of your questions and give you a copy of this form to keep for yourself.

This form provides important information about what your child will be asked to do during the study, about the risks and benefits of the study, and about your child's rights as a research participant.

- If you have any questions about or do not understand something in this form, you should ask your child's teacher for more information.
- You should feel free to discuss participating with anyone you choose, such as family or friends, before you decide to allow your child to participate.
- Do not agree to participate in this study unless your child's teacher has answered your questions and you decide that you want your child to be part of this study.
- Your child's participation is entirely voluntary, and you can refuse to participate or withdraw at any time.

Title of Research Study: Strategies to develop effective problem-solving habits for English Learners in a Problem-Based Learning Classroom

Student Researcher and email address: Ian Dove Lempke, ian.lempke@[redacted]

Faculty Advisor, Hamline affiliation/title, phone number(s), and email address: James Brickwedde, School of Education, [redacted], jbrickwedde@[redacted]

1. What is the research topic, the purpose of the research, and the rationale for why this study is being conducted?

Mr. Lempke is studying how students solve math word problems, especially students who speak more than one language. He is testing out different strategies to help students solve word problems and wants to see how students try to solve word problems after learning the strategies.

2. What will your child be asked to do if they decide to participate in this research study?

Your child will not be asked to do anything different from the other students in the class. They will still do the same work as everyone else and learn the same things, but Mr. Lempke will make a copy of some of their assignments to try to understand their thinking better. These assignments will be three daily warm ups, three Flipgrid videos where your child explains how they solved a problem, three of their unit tests, and three short surveys about how your child feels about problem solving.

3. What will be your child's time commitment to the study if they participate?

The study will take 8 weeks. Your child will not have to spend any extra time on the study.

4. Who is funding this study?

The study is not being funded.

5. What are the possible discomforts and risks of participating in this research study?

Mr. Lempke will make every effort to protect your child's privacy. However, there is a small chance that somebody else working on the study might recognize your child's work.

In addition, there may be risks that are currently unknown or unforeseeable. Please contact me at ian.lempke@[redacted] or my faculty advisor James Brickwedde at [redacted] or jbrickwedde@[redacted] to discuss this if you wish.

6. How will your child's privacy and the confidentiality of your data and research records be protected?

Mr. Lempke will remove your child's name from everything they turn in for the study to keep their identity safe. Mr. Lempke will use a fake name for your child when writing about their classwork.

Mr. Lempke also will only use their words from any Flipgrid videos they do, not the actual video.

7. How many people will most likely be participating in this study, and how long is the entire study expected to last?
A total of 10-15 students will participate in the study. The study will take 8 weeks.

8. What are the possible benefits to your child and/or to others from their participation in this research study?
Your child may learn some helpful new ways to learn math and solve and understand word problems if they participate in the study. Participating will also help Mr. Lempke understand what teaching strategies work best to help all the students in the class learn. That helps everybody, including your child.

9. If your child chooses to participate in this study, will it cost you anything?
The study does not cost you or your child anything.

10.Will your child receive any compensation for participating in this study?
There is no compensation for this study because the participants will not be doing anything different from the other students.

11.What if you decide that you do not want your child to take part in this study? What other options are available to you if you decide not to participate or to withdraw?
If you choose not to let your child participate, they will still be completing all the same work as the students who do participate. The only difference is that their work will not be used for the study.
Your child's participation in this study is entirely by choice. They can choose to not participate in it. If you or your child refuse, it will not affect their relationship with Hamline University, Mr. Lempke, or [redacted]. Also, if Mr. Lempke learns anything new that might change your mind about whether or not your child should participate, he will tell you.

12.How can your child withdraw from this research study, and who should you contact if you have any questions or concerns?
You are free to change your mind after we start. You may withdraw your consent and stop your child's participation in this research study at any time without
penalty or loss of benefits for which you may be entitled. If you wish to stop your child's participation in this research study for any reason, you should tell me, or contact me at ian.lempke@[redacted], or my Faculty Advisor, James Brickwedde, at [redacted]
or
jbrickwedde@[redacted] . You should also call or email the Faculty Advisor for any questions, concerns, suggestions, or complaints about the research and your experience as

a participant in the study. In addition, if you have questions about your rights as a research participant, please contact the Institutional Review Board at Hamline University at IRB@[redacted] .

13.Are there any anticipated circumstances under which your child's participation may be terminated by the researcher(s) without your consent?
No. Once your child starts participating, they can expect to keep participating unless you change your mind.

14.Will the researchers benefit from your child's participation in this study?
The only benefit Mr. Lempke will get from your child's participation is helping him learn more about helping students solve math word problems.

15.Where will this research be made available once the study is completed?
The study will be public, so people will be able to find it on the Hamline University website through the Bush Memorial Library Digital Commons. It may also be used for a presentation at a conference or published somewhere else, like in an academic journal.

16.Has this research study received approval from [redacted]?
Yes. [redacted] has given Mr. Lempke permission to complete this study.

17.Will your child's information be used in any other research studies or projects?
No - your child's information (classwork and test scores) collected as part of this research, even if identifiers are removed, will not be used in or distributed for future research studies.

Appendix F

Consent Form in Hmong

Daim Ntawv Paub Thiab Pom Rau Ib Tug Neeg Yuav Los Koom Kev Tshawb Fawb

Yuav kom koj koom nrog ib txog kev tshawb fawb. Daim ntawv no qhia rau koj txog txoj kev tshawv fawb no. Koj tus kws qhia ntawv mam li teb tag nro koj cov lus nug thiab luam daim ntawv no rau koj khws cia.

Daim ntawv no muaj lus tseem ceeb qhia txoj kev tshawb fawb no hais seb koj yuav ua dab tsi. Nws yuav qhia txog kev zoo thiab tsis zoo txog txoj kev tshawb fawb no. Daim ntawv no kuj yuav qhia rau koj txog koj txoj cai yog koj yuav los koom txoj kev tshawb fawb no.

- Yog koj muaj lus nug txog ib yam dab tsi los yog koj tsis nkag siab txog ib yam dab tsi nyob hauv daim ntawv no, ces koj mam li nug koj tus kws qhia ntawv.
- Ua ntej koj yuav koom, koj mus nrog leej twg tham txog txoj kev tshawv fawb no los tau, ib yam li koj tsev neeg los cov phooj ywg.
- Koj yuav tsum tsis txhob yeem koom ua txoj kev tshawv fawb ntawm no, yog koj tus kws qhia ntawv teb tsis tau tag nro koj cov lus nug.
- Your participation is entirely voluntary, and you can refuse to participate or withdraw at any time.
- Nyob ntawm koj yeem koom ua txoj kev tshawv fawb no; yog koj tsis yeem los yog koj koom lawm tab sis koj ho pauv siab, xav taum los koj yeej tawm tau.

Txoj Kev Tshawb Fawb No Hu Li Cas: Cov tswv yim zoo los pab menyuam kawm Lus Askiv tsim txoj kev daws teeb meem hauv lub hoob kawm ntawv qhia txog cov kev txawj

Kws Tsawb Fawb thiab email: Ian Dove Lempke, ian.lempke@[redacted]

Tus Kws Qhia Ntawv saib xyuas tus kws tsawb fawb no, Nws qhov chaw ua hauj lwm/lub npe, xov tooj, thiab email: James Brickwedde, Assistant Professor of Education, [redacted], jbrickwedde@[redacted]

1. **Kev tshawb nrhiav lub npe hu ua li cas, lub hom phiaj los ntawm kev tshawb nrhiav no, thiab qhov laj thawj no yog dab tsis?**

Nai Khu Lempke kawm txog thiab xav paub seb cov menyuam kawm ntawv daws cov lus teeb meem ua lej lis cas, uas cov nws xav paub txog tshaj yog cov menyuam hai ntau yam lus. Nws yuav sim ntau txoj tswv yim los pab menyuam kawm ntawv daws cov lus teeb meem no thiab nws xav paub seb cov menyuam kawm ntawv yuav daws cov lus teeb meem no li cas tom qab uas nws qhia lawv cov tswv yim no.

2. **Yog koj txiav txim siab los koom nrog txoj kev tshawb fawb no ces koj yuav tsum ua dab tsis?**

   Koj yuav tsis ua ib yam dab tsi txawv ntawm lwm cov menyuam kawm ntawv li. Koj yeej ua koj txoj hauj lwm hauv hoob kawm ntawv ib yam li cov menyuam kawm ntawv sawv daws hauv lub hoob, tab sis Nai Khu Lempke yuav muab koj ib txhia ntaub ntawv luam los saib kawm txog koj txoj kev xav.

3. **Koj yuav siv sij hawm ntau npaum li cas, yog koj txiav txim siab los koom nrog txoj kev tshawb fawb no?**
   Txoj kev tshawb fawb no yuav mus txog 8 lub lim tiam (vas thiv). Koj yuav tsis siv sij hawm ntau tshaj qhov no rau txoj kev tshawb fawb no.

4. **Leej twg yuav muab nyiaj txiag los them txoj kev tshawb fawb no?**
   Txoj kev tshawb fawb no yuav tsis siv nyiaj txiag li.

5. **Puas muaj kev raug mob los txhawj xeeb los rau koj yog koj yeem koom ua qhov kev tshawb fawb no?**
   Tej zaum muaj leej twg nrhiav yuam kev tau koj cov ntawv xeem, los cov ntaub ntawv thiab yees duab kawm, es lawv yuav paub hais tias yog koj li no. Tab sis, Nai Khu Lempke mam ua twb zoo tiv thaiv kom tsis muaj qhov teeb meem no.

   Tsis tag li ntawv, tej zaum yuav muaj lwm tej teeb meem uas tseem tsis tau tshwm los muaj thiab. Thov tiv tauj kuv ntawm ian.lempke@[redacted] los tus kws saib xyuas James Brickwedde ntawm [redacted] los jbrickwedde@[redacted] yog koj xav muaj lus tham ntxiv.

6. **Peb yuab tiv thaiv tsis pub lwm tus neeg paub txog koj cov ntaub ntawv li cas?**
   Nai Khu Lempke mam li muab koj lub npe lwv ntawm tag nro koj cov ntaub ntawv kom tsis muaj leej twg paub hais tias yog koj. Nai Khu Lempke mam li siv npe cuav rau koj thaum nws sau txog cov tshwm sim. Nai Khu Lempke yuav siv koj lub suab xwb, tsis siv koj cov yees duab es thiaj li tsis pom koj lub ntsej muag.

7. **Yuav muaj pes tsawg leej neeg koom los ua txoj kev tshawb fawb no, thiab yuav siv sij hawm npaum li cas?**
   Yuav muaj 10-15 tus mesnyuam kawm ntawv los koom ua txoj kev tshawb fawb no. Txoj kev tshawb fawb no yuav mus txog 8 lub lis piam (vas thiv).

8. **Koj txoj kev koom qhov kev tshawb fawb no yuav pab tau koj los lwm tus li cas?**
   Tej zaum koj yuav kawm tau lwm txoj kev tshiab los ua lej thiab daws lus teeb meem, yog koj koom nrog txoj kev tshawb fawb no. Koj txoj kev koom yuav los pab tus Nai Khu Lempke qhia tau ntawv zoo dua qhov qub. Tej zaum nws yuav txawj qhia koj dua yog koj cia nws siv koj cov ntaub ntawv mus kawm.

9. **Koj puas tau them nqi los puas muaj kev ntshai, yog koj yeem koom nrog txoj kev tshawb fawb no?**
   Koj tsis them nqi los koom nrog txoj kev tshawb fawb no. Tej zaum muaj leej twg nrhiav yuam kev tau koj cov ntawv xeem xwb, tab sis Nai Khu Lempke mam ua twb zoo tiv thaiv kom tsis muaj qhov teeb meem no.

10. **Koj puas yuav tau nyiaj los khoom plig yog koj koom nrog txoj kev tshawb fawb no?**
   Rau txoj kev tshawb fawb zaum no, koj yuav tsis tau nyiaj los yog txais ib qho dab tsi li vim txhua tus yeej tsis tau dab tsi.

11. **Yuav ua li cas, yog koj tsis xav koom ua txoj kev tshawb fawb no? Koj xaiv ua lwm yam puas tau? Yuav ua li cas yog koj pib koom ua txoj kev tshawb fawb no tab sis koj hos xav tawm?**
   Nyob ntawm koj yeem koom txoj kev tshawv fawb no xwb. Koj tsis yeem koom los tau. Yog koj tsis yeem koom los yuav tsis puas koj txoj kev sib raug zoo nrog Hamline University, Nai Khu Lempke, thiab tsev kawm ntawv Hmong College Prep Academy. Tsis tag li ntawv, yog Nai Khu Lempke ho paub txog ib yam dab tsis tshiab tsim nyog ua tau koj hloov siab txog seb koj puas yuav koom los tsis koom ua txoj kev tshawb fawb no los nws mam li qhia rau koj.

12. **Yuav ua li cas yog koj tsis xav koom los xav tawm ntawm txoj kev tshawb fawb no? Yog koj muaj lus nug los muaj kev txhawj xeeb txog txoj kev tshawb fawb no, koj yuav hu rau leej twg?**
   Tab txawm koj pib txoj kev tsawb fawb lawm es koj ho hloov siab tsis xav koom los tau. Koj tawm thaum twg lo tau. Koj yuav tsis mag npluas los yuav poob dab tsis li. Yog koj ho xav tawm no los qhia rau kuv ntawm ian.lempke@[redacted], los hu rau kuv tus nai khu saib nyuas, James Brickwedde, ntawm [redacted] los jbrickwedde@[redacted]. Yog koj muaj lus nug, lus txhawj xeeb, tswv yim, los lus tsis zoo siab txog txoj kev tshawb fawb no, ces hu rau kuv tus nai khu saib xyuas thiab. Tsis tag li ntawv, yog koj muaj lus nug txog koj txoj cai ua tus neeg koom nrog txoj kev tshawb fawb no, thov hu rau Institutional Review Board hauv Hamline University ntawm IRB@[redacted].

13. **Puas muaj ib yam teeb meem ua nai khu yuav tau xaus tus neeg txoj kev koom es yuav tsis qhia nws niam thiab txiv los tus saib xyuas nws?**

   Tsis muaj. Thaum koj pib koom lawm ces, koj yeej koom kom ntxog thaum kawg, yog koj ho tsis xav tawm.

14. Tus kws tshawb fawb no puas tau ib yam dab tsis los ntawm nws txoj hauj lwm tshawb fawb no?

Koj txoj kev koom nrog txoj kev tshawb fawb no yuav los pab Nai Khu Lempke qhia menyuam kawm ntawv daws lus teeb meem thaum lawv ua lej kom zoo tshaj yav tom ntej no.

15. Lub sij hawm uas txoj kev tshawb fawb no xaus lawm, cov tshwm sim txog txoj kev tshawb fawb no yuav nyob rau qhov twg?

Txoj kev tshawb fawb no yuav muab tau rau pej xeem. Leej twg nrhiav los tau hauv Hamline University lub website, hauv Bush Memorial Library Digital Commons. Cov tshwm sim no kuj siv tau los ua kev nthuav qhia ntawm tej rooj sib tham los muab luam tau rau pej xeem, ib yam li hauv tej phau ntawv txog kev kawm.

16. Tsev Kawm Ntawv [redacted] puas tau pom zoo ua txoj kev tshawb fawb no?

Tsev Kawm Ntawv [redacted] pom zoo tso cai rau Nai Khu Lempke ua txoj kev tshawb fawb no.

17. Puas siv cov tshwm sim txog txoj kev tshawb fawb no rau hauv lwm cov kev tshawb fawb los lwm tej yam num?

Yuav tsis muab cov tshwm sim txog txoj kev tshawb fawb no rau lwm tus neeg siv rau lwm cov kev tshawb fawb tom ntej no. Tab txawm koj lub npe tsis nyob qhov twg kom lwm tus neeg pom los yuav tsis pub lwm tus neeg siv koj cov ntaub ntawv kawm thiab qhab nia xeem ua yav tom ntej no.

Appendix G

Consent form in Karen

Hamline University
Institutional Review Board has approved this consent form.
IRB approval # 2019-08-51E
Approved: 09/02/2019
Expires five years from above approval date.

[illegible]

[illegible]

[illegible]

- [illegible]
- [illegible]
- [illegible]
- [illegible]

[illegible] – [illegible]

[illegible] e-mail address– Ian Dove Lempke [illegible] ian.lempke@[redacted]

[illegible], Hamline [illegible], [illegible] phone [illegible] e-mail address– James Brickwedde, School of Education, [redacted] jbrickwedde@[redacted]

1. **[illegible]?**

   Mr. Lempke [illegible]

2. **[illegible]?**

   [illegible] Mr. Lempke [illegible]

   [illegible] 3 [illegible] 3 [illegible]

ဖဲလၢ Mr. Lempke ဟ့ၣ်လီၤနဖိလံာ်သ့ၣ်တဖၣ်ဖဲအဝဲဒိးန့ၣ်ကူၤဝဲအဆၢကတီၢ်အဝဲကထုးသံကွၢ်ဝဲနဖိအမံၤဒီးကဟ့ၣ်လီၤနဖိအမံၤလိၣ်ကွဲဖဲလၢ Mr. Lempke ကွဲးဘၣ်ဃးနဖိဟံၣ်တၢ်မၤန့ၣ်လီၤ. Mr. Lempke ကသူလံာ်မဲဖျၢလၢဟဲလၢ Flipgrid တၢ်ဂီၤမူ,တၢၤမ့ၢ်လၢတၢ်ဂီၤမူနီၣ်ကီၣ်ဘၢ.

၇. **တၢ်ယုထံၣ်သ့ၣ်ညါတခါအံၤပှၤကွိဖိကအိၣ်ပှဲၤဂၤလဲၣ်ဒီးတၢ်ဆၢကတီၢ်ကယံၤထဲလဲၣ်?**

ပှၤကွိဖိကအိၣ်ဖဲတဲၤလၢ 10-15 ဂၤ ဒီးကယံၤဝဲ 8 နွံန့ၣ်လီၤ.

၈. **တၢ်အဘျုးအဖိၣ်အိၣ်လၢနဖိအဂီၢ်ဒ်လဲၣ်ဒီးပှၤကွိဖိအဂၤအဂီၢ်ဒ်လဲၣ်?**

တၢ်ယုထံၣ်သ့ၣ်ညါတခါအံၤဘၣ်သ့သ့ၣ်အဘျုးအဖှိၣ်အိၣ်လၢနဖိအဂီၢ်လၢကမၤစၢၤနဖိအဟံၣ်တၢ်မၤန့ၣ်လီၤ. Mr. Lempke စ့ၢ်ကီးကနၢ်ပၢၢ်အါထီၣ်ဘၣ်ဃးဒီးကျဲလၢအဂ့ၤကတၢ်လၢကမၤဆၢၤကွိဖိအဟံၣ်တၢ်မၤန့ၣ်လီၤ.

၉. **နမ့ၣ်တူၢ်လိာ်တၢ်မၤလိတခါအံၤန့ၣ်နကဘၣ်ဟ့ၣ်အဘူးအလဲတမံၤမံၤဓါ?**

နတဘၣ်ဟ့ၣ်လီၤတၢ်အဘူးအလဲဘၣ်.

၁၀. **ပှၤဟ့ၣ်ထွဲနဖိတၢ်တမံၤမံၤစ့ၢ်ဓါ?**

လၢတၢ်မၤလိတဘျီအံၤတၢ်တဟ့ၣ်နီၤလီၤခွဲၣ်ဖးနီတမံၤဘၣ်မ့ၢ်လၢကွိဖိဒီးန့ၢ်တၢ်မၤလိတမံၤယီလီၤ

၁၁. **တၢ်ယုထံၣ်သ့ၣ်ညါတခါအံၤနမ့ၢ်သးတလီၤတလိၤဘၣ်န့ၣ်နကမၤတၢ်သ့မနၢ်လဲၣ်?**
အဒိနဖိပၣ်ယုးလၢတၢ်ယုထံၣ်သ့ၣ်ညါအံၤဘၣ်ဆၣ်မ့ၢ်အဲၣ်ဒိးဟးထီၣ်ဆိန့ၣ်နဘၣ်မၤမနုာ်လဲၣ်?

နဖိမ့ၢ်တစဲးလီၤအမံၤလၢတၢ်မၤလိတဘျီအံၤသနၥ်ကူကွိဖိကီးဂၤဒၢးကမၤလိတၢ်တမံၤယီလီၤ. တၢ်တမံၤလၢအလီၤဆီတခါသၣ်တဟံး ဃးအလံာ်အလဲၢ်ဘၣ်လီၤ.

တၢ်ယုထံၣ်သ့ၣ်ညါတခါအံၤနသးမ့ၢ်တလီၤပလိၤဘၣ်န့ၣ်အတဘၣ်တၢ်နီၤတမံၤဘၣ်. ဒီးအဝဲတမၤဘၣ်ဒိးနတၢ်ဘၣ်ထွဲလိၢ်သးဒီး Hamline University မ့ၤဂ့ၤ Hmong College Prep Academy န့ၣ်လီၤ. နဖိအကြၢးဒီးတကြၢးပၣ်ယှၢ်ဒီးတၢ်ယုသ့ၣ်ညါတခါအံၤန့ၣ် Mr. Lempke ကစံးကျိၢ်နၤန့ၣ်လီၤ.

၁၂. **အဒိနဖိပၣ်ယုးလၢတၢ်ယုထံၣ်သ့ၣ်ညါတခါအံၤ, ဘၣ်ဆၣ်နမ့ၢ်အဲၣ်ဒိးဟးထီၣ်ဆိန့ၣ်နကဘၣ်မၤဒ်လဲၣ်?**
နတၢ်သံကွၢ်မ့ၢ်အဒိးတၢ်ဘၣ်ယိၣ်မ့ၢ်အိၣ်န့ၣ်နကဘၣ်စံးကျိၤမတဂၤလဲၣ်?

နခွဲးယၤအိၣ်လၢကဆီတလဲနတၢ်ဆိမိၣ်,
တချူးပစးထီၣ်ဝဲၤအလိၢ်ခံနခွဲးယၤအိၣ်လၢနကဆီတလဲနတၢ်ဆိမိၣ်သ့ဝဲဒီးပတုၤသ့ဝဲကီးဆၢကတီၢ် ဒ်းန့ၣ်လီၤ.
လၢတၢ်မၤလိဘၣ်ဃးတၢ်ယုထံၣ်သ့ၣ်ညါတခါအံၤနမံၢ်တသးလီၤပလိၤဘၣ်ဒီးကြၢးနတဲပဝဲမ့တမ့ၣ်နစံးကျိၤယၤလၢ
ian.lempke@[redacted].မ့တမ့ၢ်နကိယသရၣ် James Brickwedde 651-523-2175, jbrickwedde@hamline.edu.
ဝံသစူၤစံးကျိၤယၤမ့တမ့ၢ်သရၣ်သရၣ်မုၢ်နမ့အိၣ်ဒီးတၢ်သံကွၣ်, တဘၣ်ယိၣ်, တၢ်ဟ့ၣ်ကူၣ်ဟ့ၣ်ဖး, မ့တမ့ၣ်တၢ်ကဒုကစ့ါဘၣ်ဃၤဒီး
တၢ်ယုထံၣ်သ့ၣ်ညါတခါအံၤန့ၣ်လီၤၣ်. ဝံသစူၤစံးကျိၤ Institutional Review Board လၢ Hamline University လၢ IRB@hamline.edu
နမ့အိၣ်ဒီးတၢ်သံကွၣ်ဘၣ်ဃၤဒီးတၢ်ဂ့ၣ်တၢ်ကျိၤန့ၣ်လီၤ.

၁၃. **Mr. Lempke အမ့ၢ်မၤနဖိဟးထီၣ်လၢတၢ်ယုထံၣ်သ့ၣ်ညါတခါအံၤအပူၤ, အကွၢ်ဖဲအသးဓါ?**

အဝဲတကွၣ်ဖဲအသးဘၣ်, ဒ်သိးကွိဖိကဟးထီၣ်လၢတၢ်ယုထံၣ်သ့ၣ်ညါတခါအံၤဘၣ်ထွဲလၢကျိဖိဒီးကျိဖိမိၢ်ပၢ်အတၢ်ဘၣ်သး.

တၢ်အဆၢ, တၢ်ဒိးစဲ 3 ခါဒီးတၢ်ကွၢ်ဆိကမိဉ်ဘဉ်ယးနဖိအတၢ်တူၢ်ဘဉ်လၢတၢ်ဟ့သ့ဉ်ညါတၢ်ဂ့ၢ်ကီ.

၃. **ပှၤကိုဖိသ့ဉ်မ့ၢ်မၤလိတၢ်ကယံၤဝဲထဲလဲဉ်?**

တၢ်ဆၢကတီၢ်ကယံၤဝဲ 8 နဉ်ဒံဉ်နှဉ်လီၤ.

၄. **တၢ်မၤလိတခါအံၤမတၢၤမၤစၢၤလဲဉ်?**

ပှၤနီတဂၤဘၢ.

၅. **တၢ်ယုထံဉ်သ့ဉ်ညါတခါအံၤတၢ်ဘၢ်ယိဉ်ဘဉ်ဘှီမနုၤအိဉ်လဲဉ်?**

Mr. Lempke ကယၢကယၢနဖိတၢ်ဂ့ဉ်တၢ်ကျိၤ,ဘဉ်ဆဉ်သနာ်က့သ့ဉ်ပှၤကိုလၢအမၤတၢ်ယုထံဉ်သ့ဉ်ညါတခါမအံၤ နှဉ်အဝဲဘှဉ်တံဉ်သ့ဉ်ညါနဖိဟံဉ်တၢ်မၤန့ဉ်လီၤ.တဉ်လီၤပျဲၤလီၤဖုးလၢပတထံဉ်ဆိဒီးတသ့ဉ်ညါဘဉ်နှဉ်ကဟဲကဲထီဉ်အသးသ့နှဉ်လီၤ. ဝံသစူၤစဲးကျိၤယၢၤလၢ ian.lempke@hcpak12.org, မ့တမ့ဉ်ယသရဉ် James Brickwedde [redacted] jbrickwedde@[redacted]

၆. **ပကယုကယၢနဖိအဂ့ၢ်အကျိၤဒ်လဲဉ်?**

ဖဲလၢ Mr. Lempke ဟ့ဉ်လီၤနဖိလံာ်သ့ဉ်တဖဉ်ဖဲအဝဲဒီးနှဉ်ကူၤဝဲအဆၢကတီၢ်အဝဲကထုးသံကွၢ်ဝဲနဖိအမံၤဒီးကဟ့ဉ် လီၤနဖိအမံၤလိဉ်ကွဲဖဲလၢ Mr. Lempke ကျဲဘဉ်ယးနဖိဟံဉ်တၢ်မၤန့ဉ်လီၤ. Mr. Lempke ကသူလံာ်မဲဖျၢလၢဟဲလၢ Flipgrid တၢ်ဂီၤမူ,တၢၤမ့ၢ်လၢတၢ်ဂီၤမူနီဉ်ထီဉ်ဘၢ.

၇. **တၢ်ယုထံဉ်သ့ဉ်ညါတခါအံၤပှၤကိုဖိကအိၢ်ပှဲၤကလဲဉ်ဒီးတၢ်ဆၢကတီၢ်ကယံၤထဲလဲဉ်?**

ပှၤကိုဖိကအိဉ်ဝဲတုၤလၢ 10-15 ဂၤ ဒီးကယံၤဝဲ 8 နံနှဉ်လီၤ.

၈. **တၢ်အဘျူးအဖိဉ်အိဉ်လၢနဖိအင်္ဂါဒ်လဲဉ်ဒီးပှၤကိုဖိအဂၤအင်္ဂါဒ်လဲဉ်?**

တၢ်ယုထံဉ်သ့ဉ်ညါတခါအံၤဘဉ်သ့သ့ဉ်အဘျူးအဖှိဉ်အိဉ်လၢနဖိအင်္ဂါလၢကမၤစၢၤနဖိအဟံဉ်တၢ်မၤန့ဉ်လီၤ. Mr. Lempke စ့ၢ်ကီးကနၢ်ပၢၢ်အါထီဉ်ဘဉ်ယးဒီးကျဲလၢအဂ့ၤကတၢၢ်လၢကမၤဆၢၤကိုဖိအဟံဉ်တၢ်မၤန့ဉ်လီၤ.

၉. **နမ့ဉ်တူၢ်လိာ်တၢ်မၤလိတခါအံၤနှဉ်နကဘဉ်ဟ့ဉ်အဘူးအလဲတမံၤမံၤဓါ?**

နတဘဉ်ဟ့ဉ်လီၤတၢ်အဘူးအလဲဘဉ်.

၁၀. **ပှၤဟ့ဉ်ထွဲနဖိတၢ်တမံၤမံၤစ့ၢ်ဓါ?**

လၢတၢ်မၤလိတဘျီအံၤတၢ်တဟ့ဉ်နီၤလီၤနိဉ်ဖးနီတမံၤဘဉ်မ့ၢ်လၢကိုဖိဒီးနၢ်တၢ်မၤလိတမံၤယီလီၤ

၁၁. **တၢ်ယုထံဉ်သ့ဉ်ညါတခါအံၤနမ့ၢ်သးတလီၤတလီၤဘဉ်နှဉ်နကမၤတၢ်သ့မနုၢ်လဲဉ်?**
အဒိနဖိပဉ်ယှာ်လၢတၢ်ယုထံဉ်သ့ဉ်ညါအံၤဘဉ်ဆဉ်မ့ၢ်အဲဉ်ဒိးဟးထီဉ်ဆိနှဉ်နဘဉ်မၤမနုာ်လဲဉ်?

နဖိမ့ၢ်တစဲးလီၤအမိၤလၢတၢ်မၤလိတဘျီအံၤသနာ်က့ကိုဖိကိးဂၤဒဲးကမၤလိတၢ်တမံၤယီလီၤ. တၢ်တမံၤလၢအလီၤဆီတခါသ့ဉ်တဟးဲ ယးအလံာ်အလါဘဉ်လီၤ.

တၢ်ယုထံဉ်သ့ဉ်ညါတခါအံၤနသးမ့ၢ်တလီၤပလီၤဘဉ်နှဉ်အတဘဉ်တၢ်နီၢ်တမံၤဘဉ်. ဒီးအဝဲတမၤဘဉ်ဒိးနတၢ်ဘဉ်ထွဲလိၢ်သးဒီး Hamline University မ့တမ့ၢ် [redacted] Academy နှဉ်လီၤ. နဖိအကြၢးဒီးတကြၢးပဉ်ယှာ်ဒီးတၢ်ယုသ့ဉ်ညါတခါအံၤနှဉ် Mr. Lempke ကဝဲးကျိၢ်နၤနှဉ်လီၤ.

၁၂. **အဒိနဖိပဉ်ယှာ်လၢတၢ်ယုထံဉ်သ့ဉ်ညါတခါအံၤ, ဘဉ်ဆဉ်နမ့ၢ်အဲဉ်ဒိးဟးထီဉ်ဆိနှဉ်နကဘဉ်မၤဒ်လဲဉ်?**
နတၢ်သံကွၢ်မ့ၢ်အဒိးတၢ်ဘဉ်ယိဉ်မ့ၢ်အိဉ်နှဉ်နကဘဉ်စဲးကျိၤမတၢၤလဲဉ်?

နစဲ့းယၢၤအိဉ်လၢကဆီတလဲနတၢ်ဆိမိဉ်,
တချူးပစးထီဉ်ဝဲၤအလိၢ်ခံနစဲ့းယၢၤအိဉ်လၢနကဆီတလဲနတၢ်ဆိမိဉ်သ့ဝဲဒီးပတုၤသ့ဝဲကိးဆၢကတီၢ် ဒဲးနှဉ်လီၤ.
လၢတၢ်မၤလိဘဉ်ယးတၢ်ယုထံဉ်သ့ဉ်ညါတခါအံၤနမံၢ်တသးလီၤပလီၤဘဉ်ဒီးကြၢးနတဲပဝဲမ့တမ့ဉ်နစဲးကျိၤယၢၤလၢ
ian.lempke@[redacted], မ့တမ့ၢ်နကိယသရဉ် James Brickwedde [redacted], jbrickwedde@[redacted]
ဝံသစူၤစဲးကျိၤယၢၤမ့တမ့ၢ်သရဉ်သရဉ်မ့ၢ်နမ့အိဉ်ဒီးတၢ်သံကွဉ်, တဘဉ်ယိဉ်, တၢ်ဟ့ဉ်ကူဉ်ဟ့ဉ်ဖး, မ့တမ့ဉ်တၢ်ကဒုကဓါဘဉ်ယၢၤဒီး တၢ်ယုထံဉ်သ့ဉ်ညါတခါအံၤနှဉ်လီၤဉ်. ဝံသစူၤစဲးကျိၤ Institutional Review Board လၢ Hamline University လၢ IRB@hamline.edu နမ့အိဉ်ဒီးတၢ်သံကွဉ်ဘဉ်ယၤဒီးတၢ်ဂ့ဉ်တၢ်ကျိၤနှဉ်လီၤ.

၁၃. **Mr. Lempke အမ့ၢ်မၤနဖိဟးထီဉ်လၢတၢ်ယုထံဉ်သ့ဉ်ညါတခါအံၤအပူၤ, အကွၢ်ဖဲအသးဓါ?**

အဝဲတကွဉ်ဖဲအသးဘဉ်, ဒ်သိးကိုဖိကဟးထီဉ်လၢတၢ်ယုထံဉ်သ့ဉ်ညါတခါအံၤဘဉ်ထွဲလၢကျိုဖိဒီးကျိုဖိမိၢ်ပၢ်အတၢ်ဘဉ်သး.

၁၄. **တၢ်အဘျုးအဖှိၣ်ဟဲလၢနဖိအအိၣ်နၣ်သရၣ်သရၣ်မုၣ်ကဒိးနၣ်ဘၣ်တၢ်မနုၤလဲၣ်?**

တၢ်အဘျုးအဖှိၣ်လၢ Mr. Lempke ဒိးဘၣ်ဝဲလၢနဖိအအိၣ်မ့ၢ်ဝဲယုထံၣ်သ့ၣ်ညါဘၣ်ယးကျဲတဘိလၢကမၤစၢၤဝဲကွဲးဖိညါတၢ်ဂံၢ်တၢ်ဘါ ဒီးအတၢ်မၤလိတဖၣ်နၣ်လီၤ.

၁၅. **တၢ်ယုထံၣ်သ့ၣ်ညါတခါအံၤနမၤလိဒီးယုထံၣ်သ့ၣ်ညါဝဲလၢတၢ်လီၢ်ဖဲလဲၢ်?**

နယုတံၣ်သ့ၢ်ညါဒီးမၤလိသ့ဝဲဖဲ, Hamline University ဒီးနလဲသ့ဝဲဖဲ Memorial Liberty Digital Commons သ့နၣ်လီၤ.

၁၆. **လၢ[redacted]နှၢ်မ့ၢ်အဝဲသ့ၣ်အိၣ်ဒီးတၢ်ခွဲၤတၢ်ယၢၤဘၣ်ယး ဒီးတၢ်မၤလိတၢ်ယုထံၣ်သ့ၣ်ညါတခါအံၤဓါ?**

[redacted]Academy ဟ့ၣ်လီၤဝဲတၢ်ခွဲၤတၢ်ယၢၤလၢ Mr. Lempke ကမၤဝဲတၢ်ယုထံၣ်သ့ၣ်ညါတခါအံၤလၢကွိတဖှၢၣ်အံၤအပူၤ နၣ်လီၤ.

၁၇. **နဖိအတၢ်ဂ့ၣ်တၢ်ကျိုၤနၣ်ပှၤကသူအီၤလၢတၢ်လီၢ်အဂၤကသ့ဓါဒီးနတူၣ်လိာ်စ့ၢ်ဓါ?**

နဖိအတၢ်ဖံးလိမၤလိသ့ၣ်တဖၣ်ပတသူအီၤလၢတၢ်လီၢ်အဂၤဘၣ်နၣ်လီၤ.

Appendix H

Flipgrid Response Summaries

Initial Data Gathering

Student	Summary
Student A	Describes repeatedly subtracting 120, and says each time she does that she gets 8 pancakes, so she got 24.
Student B	Absent, never got Flipgrid.
Student C	Literally wrote script on the warm up. Describes steps procedurally to multiplying 120 by 4, then says "I use the same method to get 360".
Student D	I divide 400 by 8 and I got fifty because I know 400 milk with get 8 pancakes, (repeats self) so that's how I know my answer is right.
Student E	I divided 120 by 8 which equals 15 ml of milk equals one pancake. Then divided 400 by 15 and got 26.6 repeating. So that's pretty much it. [On paper, trial multiplication is visible].
Student F	I subtracted 400 by 120, and then I kept on subtracting until I got 40 and couldn't subtract it anymore. So I added 8 plus 8 because you get 8 pancakes (points to text in problem) and so I got 24 pancakes.
Student G	Summarizes problem first. So what I did was I added 120 until I got 360, and if I add it one more time it will go over 400 but I don't want that so I look at how many times I added it, which is 3, and then I multiplied it by 8, which is how I got my answer. And then I subtracted 360 with 400 and I found out how much he had left over.

Student H	I think the answer is 218 because I used times and minus or others but I rather use times and minus, so my answer was 218 or 30 because I got it I don't know why. [Paper shows 400-120=380, then 380/8=30).
Student I	Reads problem off paper first. And then I did 120 + 120 and it equal to 240 and 240 plus 120 which equal to 360 and then I just kept adding and it equaled to 32 pancakes.
Student J	Describes the 120 to 8 ratio. So I multiplied 120 by 3 and it gave me 16 pancakes. And then I saw I had 40 ml of pancakes left, and that would give me 3 pancakes. And so you would be able to make 19 pancakes. [Last part not on paper].
Student K	So I just added 120 4 times with gave me 480… "M L" of milk, so it'll be 32 pancakes. And I subtracted 80 by 480 and then I subtracted half of 8 pancakes from 32 pancakes and then got 28 pancakes, so James will be able to make 28 pancakes.
Student L	What I did was I added 120 all over again and I got… 360? I added it by 20 and then I got 140 [paper says 400]. And then I added 24 and 15 and got 39. I divided 120 and 8 and got 16 [paper says 15] and that's how I got it. [paper has no answer].
Student M	What I did was I was adding it and I decided not to… (points to where his repeated addition reached 480 and where he erased the last 120). So I added it, and I decided to put them into circles, like 20, 20 (etc). So James can make 3 more pancakes. And then I divided it, but incorrectly.
Student N	So this warmup is about James and how many pancakes he can make. So there's 400 ml of milk for 2, and 120 ml of milk can make 8 pancakes. So, then I did 400 minus 120, but then I did it 3 times until I couldn't minus it anymore. So then I did 8 times 3 because 120 ml makes 8 pancakes… (ran out of time)
Student O	So, what I did was 6 times 20 is 120, so then I drew it like this (points at the circles 20s), so then I had 3, so then my answer is James can make 3 more pancakes. And then another way is I added it, and I did it 3 times, so I had 8 times 3, so I have 3.
Student P	What I did was made 120 times 3 which equals 360. Since that didn't make 400, I added 360 plus 120, which is 480. And the milk

	left is 400, so to make that I added, and it had 480, which I did here (pointing at a ratio list), where 120 times 3 equals 360, but making up 400 will do that and it equals 24, so James will make 24 pancakes.
Student Q	So first I decided to subtract 400 by 120, and I did that 3 times, and I got 40, since 120 milk equals 8 pancakes that he can make. So I multiplied 8 by 3 to get 24, and I think that's the answer for how many pancakes he can do, and there's 40 milk left.
Student R	So, what I got was 3 pancakes because I did 400, which was how much milk he had left, and then minus 120 because that's how much makes 8 pancakes, and so I just did the answer minus 120, and I got 40.
Student S	So, what I did was… James only had about… he knows that 120 ml will make 8 pancakes. And what I did was 400 divided by 120. That equaled, like, 3… 3.3 repeat, and I got 3 and 3 eighths… and, and yeah.

Midpoint Data Gathering

Student	Summary
Student A	What I did was I took out 13,000 ml to 6,000 ml and got 13 ml left, right? [Pointing to subtracting 3000 gravel AND 3000 plants, but somehow got 13,000] and then I divided by 750 and got 17.3 repeating and it would take that much to fill up the tank.
Student B	I did 13000 divided by 750, and I got that. Then I got 17 point three three three three.
Student C	[Starts by listing the information from the problem.] So I did it two ways. The first ways was I plussed 3000 with 750, and I got 3750. And then I added 13000 and I got 16750. And the other way is that I divide. So I divided 13000 by this [points to 3000] and it's just like this [points to long division, explains the long division procedurally].
Student D	So what I did, I divided my two numbers, which you can see, and I think if we divide we might get the answer. And that other number, I think we don't use it, and we divide.

Student E	I divided 3000 by 750… [long pause]… and then I did 750 times one, times 2, times 3, times 4, and when I did 750 x 4 it got me 3000, so I just said Kou can fill his jug 4 times… I think?
Student F	I added… 300 ml of gravel and 750 ml of water [paper says 3000]… and then I just kept adding 750 and I counted by the sides about how many times I added. And I got 12750 for my last one, so I decided he needs to fill up his jug 13 times.
Student G	[reading a script he wrote for himself] So the tank can hold 13,000 ml but he puts 3000 ml of gravel, so what I'm thinking is 13000, subtract 3000 is 10000, and this is the number of how much it can hold [points] right there. But his just can only hold 750, so it can only be poured in once, and you can pour it a second time, but it's going to be a fraction.
Student H	Video issue. Only 2 seconds long, just him holding up the warm up. Warm up shows subtracting 3000 from 13000, then subtracting 750 from 10,000. It just says "this is how I show my work". Fixed later: "This is how I get 10,350… in Kou's. You subtract." [Hold paper close to the camera in silence for the rest].
Student I	So what I did was, I did 750 times 14, and that got me 10,500, and I said Kou has to go 14 times to fill up his jug and fill up the tank.
Student J	So what I did was I took 13000 and subtracted it by 750 because that's how much his jug can hold. And so I did all this subtraction here [points to repeated subtraction], but then I realized I could just divide 13000 by 750, and I would get 17.33 repeating, so I said Kou could fill up his tank a total of 17 times.
Student K	Didn't upload a video yet. Paper shows a picture with a jug labeled 750 ml, and a tank with the gravel & plants drawn and labeled, followed by the water. Explanation says "I'm not for sure, but since the tank can hold 13,000 ml & since he put 3000 ml in it then if the jug can hold 750 ml of water… so he will have to fill up the jug 13 times." Scratchwork shows subtracting 13000 by 3750, then the expression 9250-(750x12) = 250.

	Came later to tell me in person. Explained what was written on paper, then said "I thought it was 13 times because I multiplied 13 by 750, which got me 9750 millimeters.
Student L	So the tank can hold… 13000 of gravel… but… uh… Kou put in… 3000 of gravel… so, what I'm thinking that 13000 – 3000 will equal 1000. It can go to once, but it can also go to twice.
Student M	Alright, so what I did was 13, take away 3,000 and I got 10,000, and then I did 10,000, take away 9,650, and I got 1359, and then I did take away 750, and the final answer is now 611. So I multiplied 750 by 15 and I got 11250, and that wasn't correct, so I did 750 times 12 and I got 7,500, and then I did 750 times 13 and got 9,650, and that was correct, and I did 750 times 14…
Student N	So today we're doing a warm up about a dude putting water in his fishtank. We're trying to figure out how many times of water does he need to fill up his tank. So what I did – I did the tank, you know, how much did it weigh, and I minused 3000 and 3000 because of the plants *and* the gravel, and after that I just minused 75 because of the water [pointing to subtracting 750 repeatedly], how much the water holds, for the jug. So I keep repeating the same thing before I can't take away 750, and I got 9 times for the times he will fill his tank with water.
Student O	So what I did was 13000-3000, which is 10,000, and then 10,000 divided by 650 [paper shows 750] was 13.3 repeating (slightly inaudible) times. So it would take him 13.3 repeating to fill up the tank. [Erased work shows lots of trial addition or trial multiplication going on.]
Student P	For this problem, my answer was that it would be 4 times. I got it because I timesed it by 4 – I did 750 times 4 – and I got 3000. [Inaudible]. So, I said it was 4 times, because the jug can only hold 750 ml, so that was my reason.
Student Q	Um, so, what I did was I divided 13,000 by 750, and I got 17.3 repeating. So, I think Kou needs to fill his tank 17.3 times.

Student R	So what I got was 13.3. How I got it – um – I did 13,000 minus 3,000, which I got 10,000, and I divided by 750, and then I got 13.3. Well, it's repeating. Okay, bye bye.
Student S	So what I did was I did 13 minus 3,000, which got me 10,000, and then I did 10,000 minus 750, which got me 9,250. And he needs to – so – he needs to do 9,250 times to fill up the tank and that's all.
Student T	Okay, so I divided these 2 number [points to 13000/750, and I got 17 point over 3, because if I divide them, it equals me, like 17 point 3, 3, 3, repeating over and over. So then I multiplied it by this number [points to x3000], and it's, um, yeah.
Student U	So first off, I did 750 times 8 [repeated addition shown on paper]. Next, I did 750 times 9, which is 6750, and the other one is 6000, so 6000 plus 6750 equals 2750 [paper shows 12750], so I did that kind of math.

Final Data Gathering

Student	Summary
Student A	Firstly I found out that 45 x 4 is 180 and then I added 5 15s because 45 divided by 3 is $15 for one hour. So then I kept repeating that and I got 15 hours Sue will have enough to buy his X-Box.
Student B	Just holds his warm up to the camera. Needs to try again. Paper shows adding 3 repeatedly, each with an arrow pointing to 45. He does this 5 times, and then adds up the 45s to get 225. Then he adds 31 to get 299? Writes 15 hours. Redid the video after break: Begins by reading the problem verbatim. "So, what I did was I wrote 3s for the hours and 45 for the money. I just keep adding up the 45s and the hours and see how I can get to this (pointing at number, can't see clearly), and I got it to 15 hours and it was $225. Plus his money – plus his $31 he got saved up. That means it's enough to buy his Xbox. And what I felt about this problem was I felt good because it was pretty easy to me and yeah I can explain to others in my class.

Student C	So what did I do was that [inaudible] would cost him $249, and he saved up $31, and then they would pay him $45 for babysitting for 3 hours. So then I did, I [checked?] them like 2 times, like 249 minus 31 equals 218, and then I minus 45 with 218 and that equals to 233. And then I put the 3 hours in there too, and then it equals to 230, and then I do it 2 times [i.e. repeats the same subtraction again] and it [added to?] is 20, and he need 20 hours to babysit to get enough money to buy the xbox.
Student D	How I got my answer is… because… I… added 31 + 45 which equals 76 dollars, and then I added 45 and 45 again until it makes the right amount of money, and then I got six 3s, and then I multiplied 3 times 6 and got 18 hours, so he needs 18 hours till he get the right amount of money.
Student E	So for this one, what I did was divide 45 by 3, which got me 15, and 1 hour equals $15. Then I multiplied 45 and 4 and 3 hours and 4, which got me 180 and 12. After getting 180, I added 31 and 180, which got me to 211, and then I added 30, which is 2 hours, $30. Then, that got me 241. Then, that's as close as I can get, so I said it would take Sue 14 hours.
Student F	I added 45 + 31 and I got 76 for… so far. And then I did 249 as the total and I subtracted 76 and I got 173 left – more money needed. Then I did 45 divided by 3 equals $15 per hour and I multiplied 15 by random numbers and I got 15 times 16 equals 244 which is way to small so I decided to do 15 times 17 which equals to 255 and he could get some more leftover money left. And then he will need to babysit 17 more hours to be able to get the Xbox.
Student G	So this is my… um… warm up. [Inaudible]. So I, um, labeled everything, like what he saved up. I did 249 subtract by 31 and I got 218, and I did 31 divided by 218 and I got 7.0322, and there's more but I didn't want to say all of it [referring to repeating decimal]. And then I did 3 hours times the total I got 21 hours, and then there was more so I got 9 minutes 67 seconds, and… more [referring to the milliseconds].
	[On other side] Mine was kinda sad because I didn't really know how to explain, and word problems are hard.

Student H	So this is how I did my… I did 31 times 45, then I did 145 [the product he got] minus 249, so that's 244 so 31 Sue earn each day, then Sue gets 45 a month. [Flips paper and explains survey answers].
Student I	So what I did was 31 plus 45 and I got 76 plus 45 and I got 121 and I added 45 and then… it got me 166. I added 41 and, um, I got 208 plus 45 and I got 253 and I got it's 253, but it's more than that.
Student J	So the first thing I did was subtract the two numbers I was given, which was 249 and 31 dollars, and I used that to find what I would have to find using this graph [pointing at table] or this table. And I also found out that 45 divided by 3 is 15 which means he makes $15 per hour, so I just multiplied 15 by 3, 6, 9, 12 and 15. And 15… working 15 hours would earn him $225 and that would be enough to get the rest of what he needs to get his Xbox.
Student K	Sue earns $45 in 3 hours for babysitting his neighbors' kids. Then, all I did was, 45 five times and got 235 and I added 31 to 245 and got $266. And I'm not sure what I did over here [pointing to a nonsensical percentage diagram] but yeah.
Student L	So what I did was I multiplied 31 by 45… um… and then I got 55, so then I… I multiplied 4 by 31, and I got 134, and I got 189. He would get 189… oop… uh, he would need 189 more to get his Xbox.
Student M	So what I did was 15 times 16 which is 240 and then I wrote… um, um… 15 times 16, um is the closest, so in 16 hours he will get $240 and he just need 9 more dollars to get his Xbox. Kbye.
Student N	So today's problem is about Sue trying to buy an Xbox and we're trying to find how many hour he needs so he can buy the Xbox that he wants. So then, um, I did 45 and - how I got 45 was I got it from the question/problem which says $45, that equals to 3 hours, and um, I did 45 times 4 because I thought that was the closest, and since $45 equals to 3 hours, I did 3 to the power of 4 and got 81 hours. After that I added – and then I got 215 – so I added $45 which got me 3 more hours and then I got – um, yeah. And I got… that number. And then after that I added his savings which got me 246, and after that I divided 45 by 3 because it says how many – because it says the Xbox is 249 and – [video cut off before describing trying to find how much he gets in 15 minutes].

Student O	Okay, so I what I did was 45 times 5 which gives me 225 and I did 225 plus 31 – 31 was his savings – and it gave me 256. Um, the Xbox costs 249, and then when I did 45 times 5 I did 3 hours times 5 which gave me 15 hours, and, so Sue will have to babysit 15 hours to have enough money to buy the Xbox.
Student P	So what I did with this problem was I divided $45 with 3 hours and I got 15, and I kept adding 15 until I reach two hundred – or I *try* to reach two hundred and forty-nine dollars. And then I multiplied 15 times 17 – I got 255, which is extra money for… for… Sue to buy… the Xbox. And… with the rest of the money he could [inaudible]. So basically I found $255, so it's 17 hours until he gets his Xbox.
Student Q	So what I did was, I drew the Xbox which was $249. And the thing I got – it says he got $31 and I did – since he says that… he remembers that last time they pay him $45 for babysitting for 3 hours– so 31 [plus] $46 and that's 76 and then subtracted 249 and 76 to get 173. And I made a table, since 3 hours, $45, I divided 45 by 3 and I got 15, and that means 1 hour gets $15, and I divided 173 by 15 and I got 11.53 repeating.
Student R	So what I did is, since he had, since the Xbox cost 248 – 249 actually – and he had $31, and last time he got paid for the babysitting, it was $45 for 3 hours, and the Unit rate is 15 over 1 because if you do 45 divided by 3 you get 15 over 1 – the 1 is the hours, so like, and then… [video ends. Paper shows multiplying 15 by 15 and adding 31 to get 256].
Student S	So what I did was I did 45 plus 31, which got me 76, and I tried adding 45 dollars to get close to 249, which is what he needs for the Xbox. I did 76 plus 45 and then 45 again and then 45 again, and I got 211. And I did 15 times 3 which got me 45, then 3 45s, that got me 9 hours, and I put 9 hours and 15 minutes, and that's how I did it.
Student T	Okay, thus um, so what I did is got he saved up $31, and the last time he remembered he got paid for 3 hours for 45… $45. And what I did was I added 31 to $45, and for 4 days (points to repeatedly adding 45) and he babysits for 3 hours and he would get the same amount as 45, and so I think he would have the money to buy an Xbox [points at 4 days].

Student U	So first I did, uh, 45 plus 45 plus 45 plus 45 [paper shows 1 more 45] plus 31. I got that 45 from $45 for babysitting 3 hours, and 3 times 15 equals 45, and [sound of timer going off] – fortyfivefortyfivefortyfivefortyfive I got 225 plus 31 which is [what he has remaining?] and that's 256 and I did a table. Here's the table. There you go – bye!